QUIZ ONE

Name Key

Difficulty (increasing): 1 2 3 4 5

1. Solve the initial value problem below. (3 pts)

$$\frac{dy}{dx} = e^{-x-y-2}, \qquad y(0) = -2$$

$\int e^y\,dy = \int e^{-x-2}\,dx \implies e^y = -e^{-x-2} + C$

$e^{-2} = -e^{-2} + C \implies C = 2e^{-2}$

$\boxed{e^y = -e^{-x-2} + 2e^{-2}}$

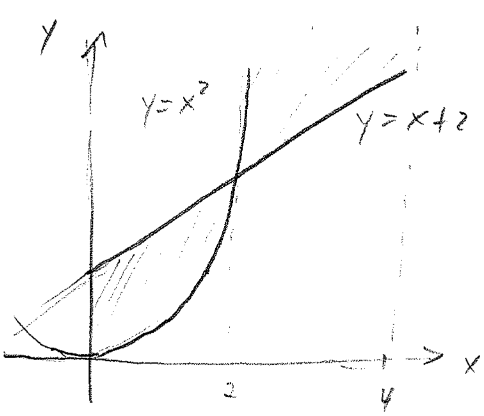
2. Which one of the following integrals corresponds to the area of the region bounded by $y = x^2$ and $y = x + 2$ from $x = 0$ to $x = 4$? (Show your work to receive full credit) (3 pts)

(a) $\int_0^4 x^2 - (x+2)dx$

(b) $\int_0^4 (x+2) - x^2 dx$

(c) $\int_0^2 x^2 - (x+2)dx + \int_2^4 (x+2) - x^2 dx$

(d) $\int_0^2 (x+2) - x^2 dx + \int_2^4 x^2 - (x+2)dx$



$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0 \qquad x = 2, -1$

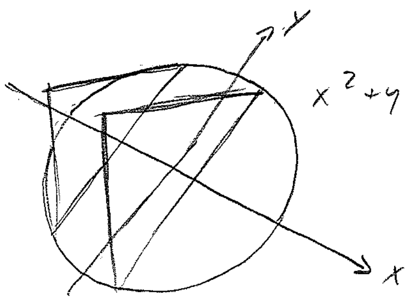
$\int_0^2 ((x+2) - x^2)\,dx + \int_2^4 (x^2 - (x+2))\,dx$

3. Does $\cosh x$ grow faster, slower, or at the same rate as e^x as $x \to \infty$? Give reasons for your answer. (3 pts)

$$\lim_{x\to\infty} \frac{\cosh x}{e^x} = \lim_{x\to\infty} \frac{\frac{1}{2}(e^x + e^{-x})}{e^x} = \lim_{x\to\infty} \frac{1 + e^{-2x}}{2} = \frac{1}{2}$$

$\therefore$ Grow at the same rate

4. Find the volume of the solid that lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross-sections perpendicular to the x-axis are triangles whose bases run from the semicircle $y = -\sqrt{1-x^2}$ to the semicircle $y = \sqrt{1-x^2}$. The height of each triangle is half the length of its base. (6 pts)

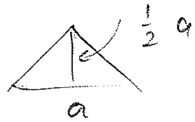


$x^2+y^2=1$

Length of base

$$l = \sqrt{1-x^2} - -\sqrt{1-x^2}$$
$$= 2\sqrt{1-x^2}$$

Area of each triangle



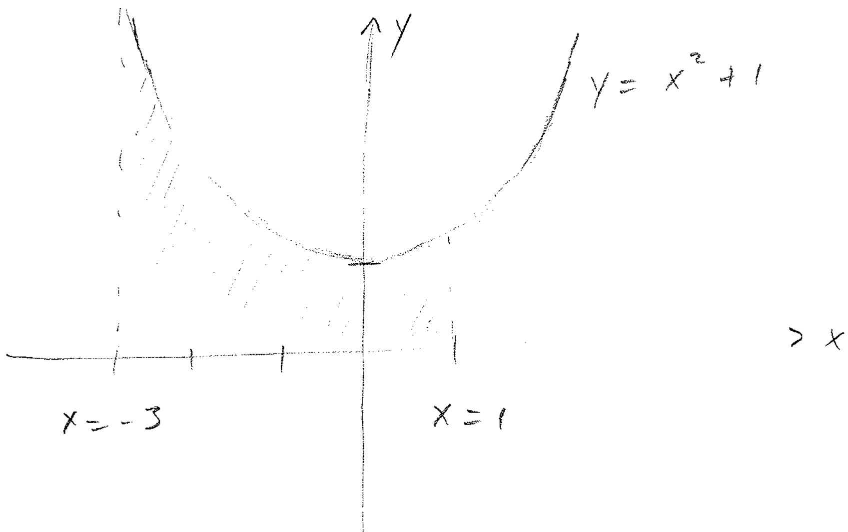
$$A = \frac{1}{2}bh = \frac{1}{4}a^2$$

$$A(x) = \frac{1}{4}\left(2\sqrt{1-x^2}\right)^2 = 1-x^2$$

$$\int_{-1}^{1} (1-x^2)\,dx = x - \frac{1}{3}x^3 \Big|_{-1}^{1} = \left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right)$$

$$= 2\left(\frac{2}{3}\right) = \frac{4}{3}$$

5. Set up an integral to find the volume of the solid generated by revolving the region bounded by $x = -3$, $x = 1$, $y = 0$, and the parabola $y = x^2 + 1$ about the x-axis. Sketch a graph and label any important parts. Simplify but **do not evaluate the integral.** (5 pts)



$$V = \int_{-3}^{1} \pi [R(x)]^2 \, dx, \quad R(x) = x^2 + 1$$

$$\boxed{V = \int_{-3}^{1} \pi (x^2 + 1)^2 \, dx}$$