Worksheet 1

1. a) $\int 2\sin x \cos x \, dx = \begin{bmatrix} u = \sin x \\ du = \cos x \, dx \end{bmatrix} = \int 2u \, du$

$= u^2 + C = \sin^2 x + C$

b) $\int 2\sin x \cos x \, dx = \begin{bmatrix} u = \cos x \\ du = -\sin x \, dx \end{bmatrix} = -\int 2u \, du$

$= -u^2 + C = -\cos^2 x + C$

c) $\int 2\sin x \cos x \, dx = \int \sin(2x) \, dx = -\frac{\cos(2x)}{2} + C$

- b) $\leftrightarrow$ a) $\quad -\cos^2 x + C = -(1 - \sin^2 x) + C$

$= \sin^2 x - 1 + C = \sin^2 x + \hat{C}$

- c) $\leftrightarrow$ a) $\quad -\frac{\cos(2x)}{2} + C = -\frac{1}{2}\left(1 + \cos(2x)\right) + \frac{1}{2} + C$

$= \cos^2 x + \frac{1}{2} + C = \cos^2 x + \hat{C}$

2.) $\int \frac{e^r}{1+e^r} \, dr = \begin{bmatrix} u = 1 + e^r \\ du = e^r \, dr \end{bmatrix} = \int \frac{1}{u} \, du$

$= \ln|u| + C = \ln|1+e^r| + C = \underline{\ln(1+e^r) + C}$

3.) $\int \ln\left(y^2 (y+1)^3\right) dy = \int \left[\ln y^2 + \ln(y+1)^3\right] dy$

$= 2\int (\ln y \, dy + 3\ln(y+1)) dy$

$u = \ln y \quad dv = dy \qquad\Big|\qquad u = \ln(y+1) \quad dv = dy$

$du = \frac{1}{y}\,dy \quad v = y \qquad\Big|\qquad du = \frac{1}{y+1}\,dy \quad v = y$

$$= 2\left(y\ln y - \int dy\right) + 3\left(y\ln(y+1) - \int \frac{y}{y+1}\,dy\right)$$

$$= 2y\ln y - 2y + 3y\ln(y+1) - 3\int \frac{y}{y+1}\,dy$$

$$\Rightarrow \left[\begin{array}{l} u = y+1 \\ du = dy \end{array}\right] = 3\int \frac{u-1}{u}\,du = 3\int\left(1 - \frac{1}{u}\right)du$$

$$= 3(u - \ln|u|) + C = 3\left((y+1) - \ln|y+1|\right) + C$$

$$\text{Ans} = y\left(2\ln y - 2 + 3\ln(y+1)\right) - 3(y+1) - 3\ln|y+1| + C$$

$$= \boxed{y\left(\ln y^2(y+1)^3 - 5\right) - 3 - \ln|y+1|^3 + C}$$

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4.) $\displaystyle\int \frac{t^3+1}{t^3-t}\,dt$

$$\begin{array}{r}
1 \\
t^3 - t \enclose{longdiv}{t^3 + 0t^2 + 0t + 1} \\
\underline{t^3 \qquad\quad - t} \\
t + 1
\end{array}$$

$$\frac{t^3+1}{t^3-t} = 1 + \frac{t+1}{t(t^2-1)} = 1 + \frac{1}{t(t-1)} = 1 + \frac{A}{t} + \frac{B}{t-1}, \quad t \neq -1$$

$\Rightarrow \quad A(t-1) + Bt = 1 \quad \Rightarrow \quad A = -B, \quad A = -1, \quad B = 1$

Check: $\displaystyle 1 + \frac{-1}{t} + \frac{1}{t-1} = \frac{t(t-1) - (t-1) + t}{t(t-1)} = \frac{t^2 - t + 1}{t^2 - t} \cdot \frac{(t+1)}{(t+1)}$

$$= \frac{t^3+1}{t^3-t} \quad \checkmark$$

$\displaystyle\Rightarrow \int \left(1 + \frac{-1}{t} + \frac{1}{t-1}\right) dt = t - \ln|t| + \ln|t-1| + C$

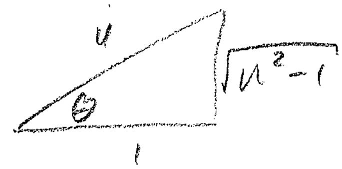
$\displaystyle = \boxed{t + \ln\left|\frac{t-1}{t}\right| + C}$

5.) $\displaystyle\int \frac{z}{\sqrt{z^4-1}}\,dz = \begin{bmatrix} u = z^2 \\ du = 2z\,dz \end{bmatrix} = \frac{1}{2}\int \frac{1}{\sqrt{u^2-1}}\,du$

$\displaystyle\begin{bmatrix} u = \sec\theta, \quad 0 \leq \theta \leq \pi/2 \\ du = \sec\theta\tan\theta \end{bmatrix} = \frac{1}{2}\int \frac{\sec\theta\tan\theta}{\sqrt{\sec^2\theta - 1}}\,d\theta$

$\displaystyle = \frac{1}{2}\int \frac{\sec\theta\tan\theta}{|\tan\theta|}\,d\theta = \frac{1}{2}\int \sec\theta\,d\theta$

$= \frac{1}{2} \ln |\sec\theta + \tan\theta| + c$



$= \frac{1}{2} \ln |u + \sqrt{u^2-1}| + C$

$= \frac{1}{2} \ln | z^2 + \sqrt{z^4 - 1}| + C$