APPM 1360 **EXAM 1 SOLUTIONS** **FALL 2008**

INSTRUCTIONS: Write your name, your instructor's name, your recitation number, and a grading table on the front of your bluebook. Start each problem on a new right-hand page. Justify your work clearly and $\boxed{\text{box}}$ your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. Text books, class notes, and crib sheets NOT permitted. Electronic devices may not be used during the exam.

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1. (15 points) Compute $\frac{dy}{dx}$ for the following functions and simplify your answers.

   (a) $y = \tanh^{-1}(\sin x)$
   $\frac{dy}{dx} = \frac{\cos x}{1 - \sin^2 x} = \frac{\cos x}{\cos^2 x} = \sec x$

   (b) $y = x \sinh^{-1}(x) - \sqrt{1 + x^2}$
   $\frac{dy}{dx} = \sinh^{-1}(x) + \frac{x}{\sqrt{1 + x^2}} - \frac{2x}{2\sqrt{1 + x^2}} = \sinh^{-1}(x)$

2. (15 points) Evaluate the following integrals and simplify your answers.

   (a) $\int \frac{dx}{\sqrt{5x^2 - 3}} = \frac{1}{\sqrt{5}} \int \frac{dx}{\sqrt{x^2 - (3/5)}} = \frac{1}{\sqrt{5}} \cosh^{-1}\left(\frac{x}{\sqrt{3/5}}\right) + C = \frac{1}{\sqrt{5}} \cosh^{-1}\left(\sqrt{\frac{5}{3}}x\right) + C$

   (b) $\int \frac{dx}{x^2 + 8x + 25} = \int \frac{dx}{x^2 + 2 \cdot 4x + 16 - 16 + 25} = \int \frac{dx}{(x+4)^2 + 9} = \frac{1}{3} \tan^{-1}\left(\frac{x+4}{3}\right) + C$

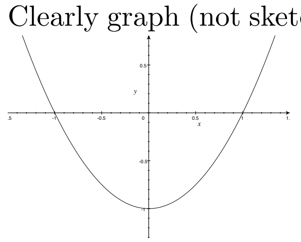
3. (25 points) Consider the solid generated by revolving about the y-axis, the region in the first quadrant bounded by $y = \cos(x^2)$ for $0 \le x \le \sqrt{\frac{\pi}{2}}$.

   (a) Calculate the volume of the object. Using shells, we get...
   $V = \int_{x=0}^{\sqrt{\pi/2}} 2\pi\, x \,\cos(x^2)\, dx = \pi \int_{u=0}^{\pi/2} \cos(u)\, du = \pi$

   (b) Set up, *but do not evaluate*, the integral calculations required to determine the surface area of the *top* of the object.
   $SA = \int 2\pi\, r\, ds = \int_{x=0}^{\sqrt{\pi/2}} 2\pi\, x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ where $\frac{dy}{dx} = \frac{d}{dx}\cos(x^2) = -\sin(x^2)2x$

4. (25 points) Consider a thin plate of uniform thickness t, density $\rho = \frac{1}{1+x}$, with a shape defined by the region in the fourth quadrant above the curve by $y = x^2 - 1$ and below the x-axis.

   (a) Clearly graph (not sketch) the region described. Be sure to find and label all intersection points.



   (b) Calculate the area of one side of the plate (the area of the region).
   $A = \int_{x=0}^{1} [0 - (x^2 - 1)]\, dx = \int_{x=0}^{1} (1 - x^2)\, dx = 2/3$

(c) Calculate the x-coordinate of the center of mass of the plate, $\bar{x}$.

In general $M_y = \int_{x=0}^{1} \tilde{x}\, dm = \bar{x} \int_{x=0}^{1} dm$.

First, note that if we use vertical strips, then $dm = \rho\, dV = \frac{1}{1+x}\, t\,(1-x^2)\, dx$ and $\tilde{x} = x$. Thus, $\int_{x=0}^{1} dm = \int_{x=0}^{1} t(1-x)\, dx = t/2$ and $\int_{x=0}^{1} \tilde{x}\, dm = \int_{x=0}^{1} x\, t(1-x)\, dx = t/6$.

Finally we see that $\bar{x} = \frac{t/6}{t/2} = 1/3$.

5. (20 points) Solve the following initial value problem:

$$\begin{aligned} \sec^2(\sqrt{y})\, \frac{dy}{dx} &= \sqrt{y} \\ y(0) &= \frac{\pi^2}{16}\,. \end{aligned}$$

Seperating variables, we get $2 \frac{\sec^2(\sqrt{y})}{2\sqrt{y}}\, dy = dx$.

Integrating each side leads to $2\tan(\sqrt{y}) = x + C$.

Applying the initial condition shows that $2\tan(\pi/4) = C$, so $C = 2$.

Finally, $2\tan(\sqrt{y}) = x + 2$ which gives $x(y)$. One could also solve for $y(x)$ if desired.