APPM 1360 **EXAM 2** **FALL 2008**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and clearly indicate your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and crib sheets NOT permitted. No electronic devices may be used during the exam.

1. (40 points) Evaluate the following integrals.

   (a) $\displaystyle\int e^{2x}\sin(3x)\,dx$

   (b) $\displaystyle\int \sqrt{25-x^2}\,dx$

   (c) $\displaystyle\int_0^4 \frac{dx}{(x-2)^4}$

   (d) $\displaystyle\int \frac{2x^2+6x-5}{(x-2)(x^2+1)}\,dx$

2. (20 points) Determine whether or not the following integrals converge or diverge. Be sure to fully explain your reasoning.

   (a) $\displaystyle\int_1^\infty \frac{x^2+5x+2}{x^4+4x^2+4}\,dx$

   (b) $\displaystyle\int_3^\infty \frac{dx}{\tanh x}$

3. (20 points) Do the following sequences converge or diverge? If the sequence converges, find the limit. Be sure to fully explain your reasoning.

   (a) $a_n = \left(1-\dfrac{2}{\sqrt{n}}\right)^n$

   (b) $a_n = \dfrac{2+n\,(-1)^n}{n!}$

4. (20 points) Determine if the following series converge or diverge. (Did we mention that you should show all of your work?) If the series converges, find the sum.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,3^n}{4^{n+1}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

THERE IS SOME USEFUL INFORMATION ON THE BACK

A short table of integrals. In the following, $a \neq 0$.

1. $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } a > 0$

2. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } u^2 < a^2$

3. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } u > a > 0$

4. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

5. $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \dfrac{1}{a}\tanh^{-1}\left(\dfrac{u}{a}\right) + C & \text{if } u^2 < a^2 \\ \dfrac{1}{a}\coth^{-1}\left(\dfrac{u}{a}\right) + C & \text{if } u^2 > a^2 \end{cases}$

6. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C \qquad \text{for } u \neq 0$

7. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C \qquad \text{for } 0 < u < a$

8. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \qquad \text{for } u^2 > a^2$

Some circular and hyperbolic trig identities.

1. $\cos^2 x + \sin^2 x = 1$
2. $\cos^2 x = \dfrac{1+\cos(2x)}{2}$
3. $\sin^2 x = \dfrac{1-\cos(2x)}{2}$
4. $\sin(2x) = 2\sin(x)\cos(x)$
5. $\cos(2x) = \cos^2(x) - \sin^2(x)$
6. $\cosh^2 x - \sinh^2 x = 1$
7. $\cosh^2 x = \dfrac{\cosh(2x)+1}{2}$
8. $\sinh^2 x = \dfrac{\cosh(2x)-1}{2}$
9. $\sinh(2x) = 2\sinh(x)\cosh(x)$
10. $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$