

A short table of integrals. In the following,  $a \neq 0$ .

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|---|---|
| 1. $\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1} \left( \frac{u}{a} \right) + C \quad \text{for } a > 0$     | 5. $\int \frac{du}{a^2 - u^2} = \begin{cases} \frac{1}{a} \tanh^{-1} \left( \frac{u}{a} \right) + C & \text{if } u^2 < a^2 \\ \frac{1}{a} \coth^{-1} \left( \frac{u}{a} \right) + C & \text{if } u^2 > a^2 \end{cases}$ |
| 2. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \left( \frac{u}{a} \right) + C \quad \text{for } u^2 < a^2$  | 6. $\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \operatorname{csch}^{-1} \left  \frac{u}{a} \right  + C \quad \text{for } u \neq 0$  |
| 3. $\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1} \left( \frac{u}{a} \right) + C \quad \text{for } u > a > 0$ | 7. $\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \operatorname{sech}^{-1} \left( \frac{u}{a} \right) + C \quad \text{for } 0 < u < a$   |
| 4. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \left( \frac{u}{a} \right) + C$                         | 8. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \left  \frac{u}{a} \right  + C \quad \text{for } u^2 > a^2$   |

Some circular and hyperbolic trig identities.

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| 1. $\cos^2 x + \sin^2 x = 1$           | 3. $\sin^2 x = \frac{1 - \cos(2x)}{2}$ | 5. $\cosh^2 x = \frac{\cosh(2x) + 1}{2}$ |
| 2. $\cos^2 x = \frac{1 + \cos(2x)}{2}$ | 4. $\cosh^2 x - \sinh^2 x = 1$         | 6. $\sinh^2 x = \frac{\cosh(2x) - 1}{2}$ |

In formulas (3)–(6),  $x$  remains fixed as  $n \rightarrow \infty$ .

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|--|---|---|
| 1. $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$ | 3. $\lim_{n \rightarrow \infty} x^{1/n} = 1, \quad x > 0$ | 5. $\lim_{n \rightarrow \infty} \left( 1 + \frac{x}{n} \right)^n = e^x, \quad  x  < \infty$ |
| 2. $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$     | 4. $\lim_{n \rightarrow \infty} x^n = 0, \quad  x  < 1$   | 6. $\lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0, \quad  x  < \infty$                     |

Here are some common Maclaurin Series and the values of  $x$  for which they converge.

- $\frac{1}{1-x} = 1 + x + x^2 + \dots + x^n + \dots = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$
- $\frac{1}{1+x} = 1 - x + x^2 - \dots + (-x)^n + \dots = \sum_{n=0}^{\infty} (-x)^n, \quad |x| < 1$
- $e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad |x| < \infty$
- $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad |x| < \infty$
- $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad |x| < \infty$
- $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{n-1} \frac{x^n}{n} + \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}, \quad -1 < x \leq 1$
- $\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| \leq 1$
- $(1+x)^m = 1 + mx + \frac{m(m-1)}{2!} x^2 + \frac{m(m-1)(m-2)}{3!} x^3 + \dots + \frac{m(m-1)(m-2) \dots (m-n+1)}{n!} x^n + \dots$   
 $= \sum_{n=0}^{\infty} \binom{m}{n} x^n, \quad |x| < 1$

$$\text{Arc length } L = \int \sqrt{1 + \left( \frac{dy}{dx} \right)^2} dx = \int \sqrt{1 + \left( \frac{dx}{dy} \right)^2} dy = \int \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} dt = \int \sqrt{r^2 + \left( \frac{dr}{d\theta} \right)^2} d\theta$$