INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) your name, (2) instructor's name, and (3) "**SUMMER 2009/TEST 1**" on the front of your bluebook. Also make a scoring table with room for 6 problems and a total score. **Work all problems. Start each problem on a new page. Clearly mark your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK.**

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1. (13 pts) Find the *centroid* of the thin plate bounded by $y = \cos(x)$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and the x-axis. (***Hint:*** Note that by symmetry we have $\bar{x} = 0$.)

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2. (13 pts) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $y = 1$ and $x = 0$ about the line $x = 4$, using the ***disk/washer method***.

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3. (25 pts) Solve the given differential equations:

   (a) $\cosh(x)\dfrac{dy}{dx} + \sinh(x)y = e^{-x}$ (b) $\dfrac{dy}{dx} = \dfrac{2x^2+1}{xe^y}$, $x > 0$

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4. (25 pts)

   (a) Set up but **do not solve** an integral (or integrals) to find the length of the curve $x = 2\sqrt{4-y}$ from $y = 1$ to $y = 4$. (*Be sure to simplify the integrand.*)

   (b) Now **find the *surface area*** of the surface generated by revolving $x = 2\sqrt{4-y}$, $1 \leq y \leq 4$ about the y-axis. (*Simplify your answer as much as possible.*)

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5. (12 pts)

   (a) Graph the region bounded by $y = x^2 - 1$ and $y = 2 - 2x^2$, $-3 \leq x \leq 3$. (*Be sure to label all relevant intersection points.*)

   (b) Now set up but **do not solve** an integral (or integrals) to find the area of the region bounded by $y = x^2 - 1$ and $y = 2 - 2x^2$, $-3 \leq x \leq 3$. (*Be sure to simplify the integrand.*)

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6. (12 pts)

   (a) Graph the region bounded by $y = 2x - 1$, $y = \sqrt{x}$, and $x = 0$. (*Be sure to label all relevant intersection points.*)

   (b) Now set up but **do not solve** an integral (or integrals) to find the volume of the solid generated by revolving the region bounded by $y = 2x - 1$, $y = \sqrt{x}$, and $x = 0$ about the y-axis, using the ***shell method***. (*Be sure to simplify the integrand.*)

THERE ARE SOME USEFUL FORMULAS ON THE OTHER SIDE!

FORMULA SHEET

Some identities

1. $\cos^2(x) + \sin^2(x) = 1$
2. $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$
3. $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$
4. $\cosh^2(x) - \sinh^2(x) = 1$
5. $\cosh^2(x) = \dfrac{\cosh(2x)+1}{2}$
6. $\sinh^2(x) = \dfrac{\cosh(2x)-1}{2}$

Inverse Trigonometric Integral Identities

1. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(\frac{u}{a}) + C,\ u^2 < a^2$
2. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$
3. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C,\ u^2 > a^2$

Inverse Hyperbolic-Trig Integral Identities

1. $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(\frac{u}{a}) + C,\ a > 0$
2. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(\frac{u}{a}) + C,\ u > a > 0$
3. $\displaystyle\int \frac{du}{a^2-u^2} = \frac{1}{a}\tanh^{-1}(\frac{u}{a}) + C,\quad \text{if } u^2 < a^2$
4. $\displaystyle\int \frac{du}{a^2-u^2} = \frac{1}{a}\coth^{-1}(\frac{u}{a}) + C,\quad \text{if } u^2 > a^2$
5. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\text{sech}^{-1}(\frac{u}{a}) + C,\ 0 < u < a$
6. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\text{csch}^{-1}\left|\frac{u}{a}\right| + C,\ u \neq 0$

Moments, Mass and Center of Mass of a Thin Rod along x-axis

Mass: $\displaystyle M = \int_a^b \delta(x)dx$ Moments: $\displaystyle M_O = \int_a^b x\delta(x)dx$ Center of Mass: $\displaystyle \bar{x} = \frac{M_O}{M}$

Moments, Mass and Center of Mass of a Thin Plate

Mass: $\displaystyle M = \int dm$ Moments: $\displaystyle M_x = \int \tilde{y}\ dm,\ M_y = \int \tilde{x}\ dm$ Center of Mass: $\displaystyle \bar{x} = \frac{M_y}{M},\ \bar{y} = \frac{M_x}{M}$