INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) your name, (2) instructor's name, and (3) "**SUMMER 2009/EXAM 3**" on the front of your bluebook. Also make a scoring table with room for 4 problems and a total score. **Work all problems. Start each problem on a new page. Clearly mark your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK.**

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1. (30 pts) Suppose $f(x) = \sum_{n=2}^{\infty} \frac{x^n}{n \ln(n)}$
   (a) For what value(s) of x is $f(x)$ absolutely convergent?
   (b) For what value(s) of x is $f(x)$ conditionally convergent?
   (c) For what value(s) of x is $f(x)$ divergent?
   (d) State the radius of convergence of $f(x)$.
   (e) Find a series representation for $f'(x)$.
   (f) Find the interval of convergence of $f'(x)$.

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2. (20 pts)
   (a) Find a Maclaurin series representation of $f(x) = x^2 e^{-x/2}$ (*You may use your knowledge of the Maclaurin Series of* e^x *to answer this question.*)
   (b) Estimate the error if we use the first **three** nonzero terms of the series found in part (a) to approximate $f(x)$ for $-1 < x < 0$.
   (c) Is the approximation used in part (b) an *underestimate* or an *overestimate*? Justify your answer.

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3. (30 pts) Determine if the given series are absolutely convergent, conditionally convergent or divergent. Find the sum of the series when possible. Justify your answer.
   (a) $\sum_{j=0}^{\infty} e^{j+1}\pi^{-j-1}$ (b) $\sum_{k=2}^{\infty} \frac{\cos(k\pi)}{k\sqrt{k^2-1}}$ (c) $\sum_{n=2}^{\infty} \ln\left(\frac{1}{n}\right) - \ln\left(\frac{1}{n^2}\right)$

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4. (20 pts) True or False (*No need to justify your answer, just write "True" or "False".*)
   (a) If $\sum |a_n|$ converges then so does $\sum a_n$.
   (b) If s_n is the sequence of partial sums of the series $\sum a_n$, and if $\lim_{n\to\infty} s_n = 0$ then the series $\sum a_n$ converges.
   (c) Suppose a_k and b_k are non-negative and $a_k > b_k$ for all k, then $\sum a_k$ converges whenever $\sum b_k$ converges.

THERE ARE SOME USEFUL FORMULAS ON THE OTHER SIDE!