NAME: Solutions

APPM 1360.300—Quiz 2: Techniques of Integration—06/18/09

Clearly print your name above. Show all work to receive full credit. Justify all answers!

1. (5 points) Evaluate $\int_0^{\pi/4} \sqrt{1-\cos(6x)}dx$

$= \int_0^{\pi/4} \sqrt{2\sin^2(3x)}\,dx$

(Trig identity)

$= \sqrt{2}\int_0^{\pi/4} \sin(3x)\,dx = -\frac{\sqrt{2}}{3}\cos(3x)\Big|_0^{\pi/4}$

$= -\frac{\sqrt{2}}{3}\left(-\frac{\sqrt{2}}{2}-1\right) = \frac{1}{3}+\frac{\sqrt{2}}{3}.$

2. (5 points) Evaluate $\int_1^e x^3\ln(x)dx$

(I.B.P.)

$u = \ln x \quad dv = x^3dx$

$du = \frac{1}{x}dx \quad v = \frac{x^4}{4}$

$= \frac{x^4}{4}\ln x\Big|_1^e - \int_1^e \frac{x^3}{4}dx$

$= \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^e = \frac{e^4}{4} - \frac{e^4}{16} + \frac{1}{16} = \frac{3e^4+1}{16}$

3. (5 points) Evaluate $\int_1^5 \frac{dx}{x^2-6x+13}$

(Complete the square and inverse trig.)

$= \int_1^5 \frac{dx}{(x-3)^2+4} = \int_{-2}^{2} \frac{du}{u^2+(2)^2}$

$u = x-3$, $du = dx$, $x=1 \Rightarrow u=-2$, $x=5 \Rightarrow u=2$

$= \left[\frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)\right]_{-2}^{2} = \frac{1}{2}\left[\tan^{-1}(1) - \tan^{-1}(-1)\right]$

$= \frac{1}{2}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \frac{\pi}{4}$

4. (5 points) Determine $\int \frac{dx}{x^2+2x}$ $= \int \frac{dx}{x(x+2)} = \int \left[\frac{A}{x} + \frac{B}{x+2}\right] dx$

(Partial fractions)

$\Rightarrow A(x+2) + Bx = 1 \Rightarrow -A = B$; $2A = 1$

$\Rightarrow A = \frac{1}{2}, B = -\frac{1}{2}$

$\Rightarrow \int \frac{1}{2}\left[\frac{1}{x} - \frac{1}{x+2}\right] dx = \frac{1}{2}\left[\ln(x) - \ln(x+2)\right] + C$

$= \frac{1}{2}\ln\left[\frac{x}{x+2}\right] + C$

Extra Credit: Show $\int \sec u du = \ln|\sec u + \tan u| + C$

(multiply by 1)

$$\int \sec u \, du = \int \sec u \cdot \left(\frac{\sec u + \tan u}{\sec u + \tan u}\right) du$$

$$= \int \frac{\sec^2 u + \sec u \tan u}{\sec u + \tan u} du$$

Let $w = \sec u + \tan u$

$dw = (\sec u \tan u + \sec^2 u) du$

$$= \int \frac{dw}{w} = \ln|w| + C$$

$$= \ln|\sec u + \tan u|$$

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