

1. (a) Find the radius of convergence of: $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$
- (b) For what value(s) of x does the series in part (a) converge absolutely?
- (c) For what value(s) of x does the series in part (a) converge conditionally but not absolutely?
- (d) For what value(s) of x does the series in part (a) diverge?
- (e) Find a series representation for $f'(x)$ and $\int f(x)dx$.
2. (a) Find the Maclaurin Series of $f(x) = \ln(1+x)$. Show all work.
- (b) Use the first three nonzero terms of the series found in part (a) to estimate $\ln(1+x)$.
- (c) Estimate the error of the approximation used in part (b) if $|x| < 0.1$.
- (d) Is the approximation used in part (b) an underestimate or an overestimate?
3. Find the *Taylor Series* of $f(x) = \sin(x)$ centered at $a = \pi/2$. (*Show all work!*)
4. (a) Use a Taylor polynomial of order $n = 2$ to find a quadratic approximation of $f(x) = (1+x)^3$ at $x = 0$.
- (b) For approximately what values of x will the error in the quadratic approximation be less than $1/1000$?
5. Determine if the given series converges conditionally, absolutely or diverges, find the sum of the series when possible, *be sure to justify your answer*:

(a) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2 2^n}{n!}$

(b) $\sum_{k=1}^{\infty} (-1)^k \frac{\ln(k)}{\sqrt{k}}$

(c) $\sum_{n=2}^{\infty} \left(\frac{2}{n^2-1} + \frac{4}{5^n} \right)$

(d) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

(e) $\sum_{n=1}^{\infty} \frac{1 + \cos^2(n)}{1 + n^5}$

(f) $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$
