NAME: Solutions APPM 1360.300

Series Worksheet (10pts)—Due Tuesday 07/08/09—Show all work.
Determine if the series converges or diverges, and state the sum of the series when possible.

1) $\sum_{n=3}^{\infty} \frac{7}{2^n}$

$$\sum_{n=3}^{\infty} \frac{7}{2^n} = \sum_{n=3}^{\infty} 7\left(\frac{1}{2}\right)^{n-3}\left(\frac{1}{2}\right)^3$$

$$= \sum_{n=3}^{\infty} \frac{7}{8}\left(\frac{1}{2}\right)^{n-3} \underset{|r|=1/2<1}{=} \frac{7/8}{1-1/2} = \frac{7/8}{1/2} = 7/4$$

(abs.) convergent Geo Series

2) $\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n^2}$

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n^2} = \sum_{n=1}^{\infty} \frac{n-1}{n^2}$$, now use LCT w/ $\sum 1/n$:

$$\lim_{n\to\infty} \frac{\frac{n-1}{n^2}}{1/n} = \lim_{n\to\infty} \frac{n^2-n}{n^2} \overset{L'H}{=} 1$$

so series diverges by LCT

3) $\sum_{j=8}^{\infty} \frac{1}{j^2+j}$

$$\sum_{j=8}^{\infty} \frac{1}{j^2+j} = \sum_{j=8}^{\infty} \frac{1}{j(j+1)} \underbrace{=}_{\text{by partial fractions}} \sum_{j=8}^{\infty} \frac{1}{j} - \frac{1}{j+1} = \left(\frac{1}{8}-\cancel{\frac{1}{9}}\right)+\left(\cancel{\frac{1}{9}}-\frac{1}{10}\right)+\cdots$$

$$= \frac{1}{8} - \lim_{j\to\infty} \frac{1}{j+1} = \frac{1}{8}$$

(abs.) Convergent Telescoping Series

4) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{5^n}$

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{5^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{5^n} = \sum_{n=1}^{\infty} \left(-1/5\right)^{n-1} \left(-1/5\right)$$

$$\underset{|r| = 1/5 < 1}{=} \frac{-1/5}{1 + 1/5} = \frac{-1/5}{6/5} = -1/6$$

(abs.) Convergent Geo series

5) $\displaystyle\sum_{k=1}^{\infty} \left(1 - \frac{3}{k}\right)^k$

Note $\displaystyle\lim_{k\to\infty} \left(1 - \frac{3}{k}\right)^k \underset{\text{by Table 8-1}}{=} e^{-3} \neq 0$ series diverges by Divergence Test.

6) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln(k)^3}{k^3}$

Note $\ln(k) < k^c$ for $k > K$

So $\ln(k)^3 < k^{3c} \Rightarrow \dfrac{\ln(k)^3}{k^3} < \dfrac{k^{3c}}{k^3} = \underbrace{\dfrac{1}{k^{3-3c}}}$

need $3 - 3c > 1$

$2 > 3c \Rightarrow c < 2/3$

Let $c = 1/3 \Rightarrow 3 - 3c = 2$, so $\dfrac{\ln(k)^3}{k^3} < \dfrac{1}{k^2}$ for $k > K$

so $\displaystyle\sum_{k=K}^{\infty} \frac{\ln(k)^3}{k^3} < \sum_{k=K}^{\infty} \frac{1}{k^2}$, so $\displaystyle\sum_{k=1}^{\infty} \frac{\ln(k)^3}{k^3}$ (abs) converges by DCT.