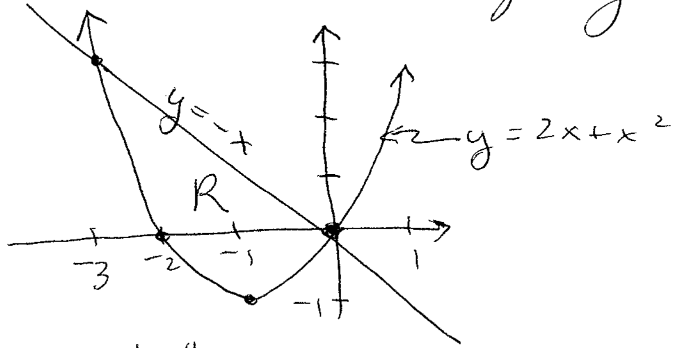
1 (a) $y = 2x + x^2 = x(2+x)$

x-intercepts: $x=0$ & $x=-2$

$y = 2x+x^2 = -x$

$\Rightarrow 3x + x^2 = 0$

intercept pts: $x=0$, $x=-3$



(b) rotate R around y-axis; use shells

$$V = \int_{-3}^{0} 2\pi \underbrace{(\text{shell radius})}_{\text{must be positive}} (\text{shell height})\,dx$$

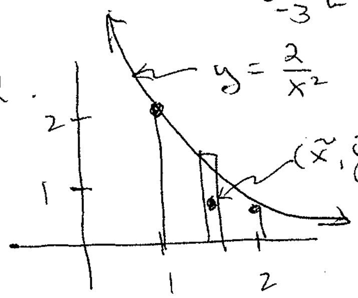
$$= \int_{-3}^{0} 2\pi\,(-x)(-x-2x-x^2)\,dx$$

$$= \int_{-3}^{0} 2\pi\, x\,(3x+x^2)\,dx$$

(c) rotate R around line $y=3$; use washers

$$V = \int_{-3}^{0} \left[\pi\,(3-2x-x^2)^2 - \pi\,(3+x)^2\right] dx$$

2.



$(\tilde{x},\tilde{y}) = \left(x, \left(\tfrac{1}{2}\right)\left(\tfrac{2}{x^2}\right)\right) = \left(x, \tfrac{1}{x^2}\right)$

height: $\frac{2}{x^2}$

width: dx

$dm = \delta(x)\cdot\frac{2}{x^2}\cdot dx = x^2\cdot\frac{2}{x^2}\,dx = 2\,dx$

$\therefore\ M = \int_1^2 dm = \int_1^2 2\,dx = 2x\Big|_1^2 = 2$

$M_y = \int_1^2 \tilde{x}\,dm = \int_1^2 2x\,dx = \frac{2x^2}{2}\Big|_1^2 = 3$

$M_x = \int_1^2 \tilde{y}\,dm = \int_1^2 \frac{2}{x^2}\,dx = -\frac{2}{x}\Big|_1^2 = -1+2 = 1$

$\therefore\ \bar{x} = \frac{M_y}{M} = \frac{3}{2}$; $\bar{y} = \frac{M_x}{M} = \frac{1}{2}$ $\Rightarrow \boxed{(\bar{x},\bar{y}) = \left(\tfrac{3}{2}, \tfrac{1}{2}\right)}$

3. $y = 20\cosh\left(\frac{x}{20}\right)$, $-20 \le x \le 20$

$y' = \sinh\left(\frac{x}{20}\right)$

$$L = \int_{-20}^{20} \sqrt{1+(y')^2}\,dx$$

$$= \int_{-20}^{20} \sqrt{1+\sinh^2\left(\tfrac{x}{20}\right)}\,dx$$

$$= \int_{-20}^{20} \sqrt{\cosh^2\left(\tfrac{x}{20}\right)}\,dx$$

$$= \int_{-20}^{20} \left|\cosh\left(\tfrac{x}{20}\right)\right| dx$$

since $\cosh\left(\frac{x}{20}\right)$ is always positive, don't need absolute value.

$= 20\sinh\left(\frac{x}{20}\right)\Big|_{-20}^{20}$

$= 20[\sinh(1) - \sinh(-1)]$

$= 40\sinh(1)$ since $\sinh x$ is odd

$= 20[e - e^{-1}]$ also acceptable.

4(a) $\int_2^4 \frac{2}{x^2-6x+10}\,dx \overset{\text{complete the square}}{=} \int_2^4 \frac{2}{(x-3)^2+1}\,dx$

$= 2\tan^{-1}(x-3)\Big|_2^4 = 2\left[\tan^{-1}(1) - \tan^{-1}(-1)\right]$

$= 2\left(\frac{\pi}{4}+\frac{\pi}{4}\right) = \boxed{\pi}$

(b) $\int \frac{x^3+4x^2}{x^2+1}\,dx$

Long division: $x^2+1 \,\overline{)\, x^3+4x^2}$ quotient $x+4$

$x^3 + x$

$4x^2 - x$

$4x^2 + 4$

$-x-4$

$= \int\left[x+4-\frac{x}{x^2+1}-\frac{4}{x^2+1}\right]dx$

$= \frac{x^2}{2} + 4x - \frac{1}{2}\ln(x^2+1) - 4\tan^{-1}x + C$

5. $\frac{dy}{dt} + t^2 y = t^2, \; y(1) = 2$

Note: Can use either method. This D.E. is separable, but can also use integrating factor method.

$P(t) = t^2, \quad Q(t) = t^2$

$v(t) = e^{\int P(t)\,dt} = e^{t^3/3}$

$y(t) = \frac{1}{v(t)}\int v(t)\,Q(t)\,dt = e^{-t^3/3}\underbrace{\int e^{t^3/3}\,t^2\,dt}_{\text{Let } u = \frac{t^3}{3}}, \; du = t^2\,dt$

$= e^{-t^3/3}\left[e^{t^3/3} + C\right] = 1 + e^{-t^3/3}C$

$y(1) = 1 + e^{-1/3}C = 2 \implies C = e^{1/3}$

$\therefore y(t) = 1 + e^{-t^3/3}e^{1/3} = 1 + e^{(1-t^3)/3}$

6(a) Given: $\left(\frac{dy}{dx}\right)^2 - y^2 = 1$ and possible solution $y(x) = \sinh x + C$

We note that for $y = \sinh x + C$

$\frac{dy}{dx} = \cosh x$

Substitute in to Diff. Eq:

$(\cosh x)^2 - (\sinh x + C)^2 = 1$

$\implies \underbrace{\cosh^2 x - \sinh^2 x}_{=1 \text{ from identity}} - 2C\sinh x + C^2 = 1$

$\implies 1 - 2C\sinh x - C^2 = 1$

$\implies -C(2\sinh x + C) = 0$

$\boxed{\therefore C = 0}$ only possibility

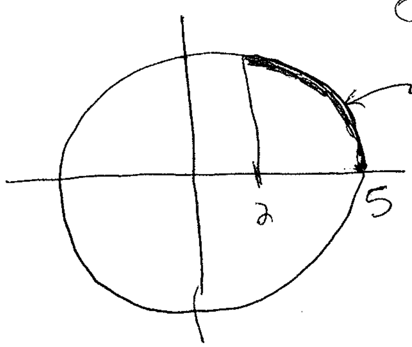
or $2\sinh x + C = 0$

$\underbrace{2\sinh x = -C}_{\text{not possible for left hand side to be constant}}$

(b) The work done by a variable force can be found by: $W = \int F(x)\,dx$. Since each of the forces are positive, the work is just the area between the force equation & the horizontal axis. By inspection, we see that $W_3 < W_1 < W_2$. Hence, the force that yields the least amount of work is F_3, followed by F_1 and then F_2.

(c) equation of circle, centered at origin, radius 5:

$$x^2+y^2=5^2$$



$y = \sqrt{25-x^2}$
rotate about x-axis
find volume using disks.

$$V = \int_2^5 \pi\left(\sqrt{25-x^2}\right)^2 dx$$

$$= \pi\left(25x - \frac{x^3}{3}\right)\Big|_2^5$$

$$= \pi\left(125 - \frac{125}{3} - 50 + \frac{8}{3}\right)$$

$$= \boxed{36\pi}$$