APPM 1360 Midterm #1 Spring 2010

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor. This exam is worth 100 points and has 6 questions.

- **Show all work!** Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes, calculators, or electronic devices are permitted.

1. (20 points) Consider the region R in the xy-plane bounded by the curve $y = 2x + x^2$ and the curve $y = -x$.
   (a) Graph the region R.
   (b) Set up but **do not evaluate** the integral to find the volume generated by revolving the region R about the y-axis.
   (c) Set up but **do not evaluate** the integral to find the volume generated by revolving the region R about the line $y = 3$.

2. (15 points) Find the center of mass, $(\bar{x}, \bar{y})$, of the thin plate covering the region between the x-axis and the curve $y = \dfrac{2}{x^2}$, $1 \le x \le 2$, if the plate's density at the point (x, y) is $\delta(x) = x^2$.

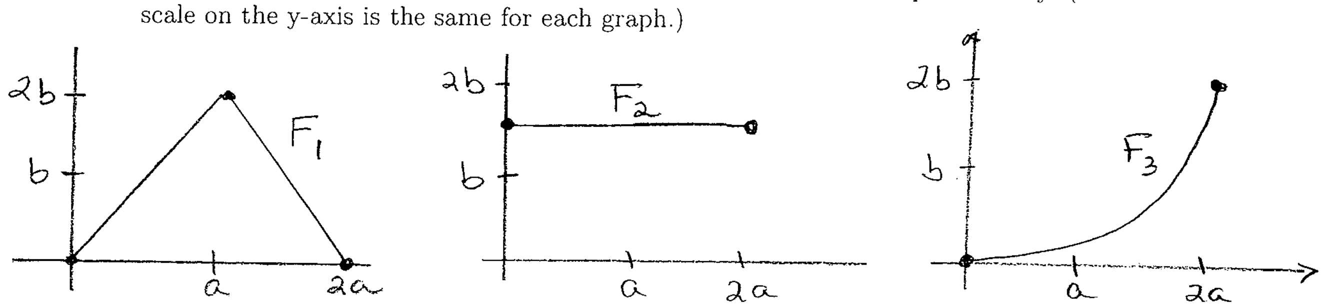
3. (12 points) An electric wire suspended between two towers forms a curve modeled by the equation $y = 20\cosh(x/20)$. Find the length of the suspended cable for $-20 \le x \le 20$. (Comment: This curve is called a catenary.)

4. (16 points) Find the requested information.
   (a) $\displaystyle\int_2^4 \frac{2}{x^2 - 6x + 10}\, dx$
   (b) $\displaystyle\int \frac{x^3 + 4x^2}{x^2 + 1}\, dx$

5. (15 points) Solve the following initial value problem for $y(t)$.

$$\frac{dy}{dt} + t^2 y = t^2, \qquad y(1) = 2$$

6. (22 points) Short answer questions. Explain your reasoning in each case.
   (a) When does $y = \sinh x + C$ solve the differential equation $(dy/dx)^2 - y^2 = 1$? Explain.
   (b) The following three graphs show the force F (in pounds) required to move an object from $x = 0$ to $x = 2a$ along the x-axis in three different situations. Order the force functions from the one that yields the least work to the one that yields the most work. Explain briefly. (Note that the scale on the y-axis is the same for each graph.)



   (c) A hemispherical bowl of radius 5 inches is filled with water that is 3 inches deep. What is the volume of water in the bowl?

Formulas for APPM 1360, Spring 2010, Exam 1

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C,\ u^2 < a^2$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C,\ u^2 > a^2$$

Inverse Hyperbolic-Trig Integral Identities

$$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C,\ a > 0$$
$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C,\ u > a > 0$$
$$\int \frac{du}{a^2-u^2} = = \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C,\quad \text{if } u^2 < a^2$$
$$\int \frac{du}{a^2-u^2} = \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C,\quad \text{if } u^2 > a^2$$
$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C,\ 0 < u < a$$
$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C,\ u \neq 0$$

Identities for Hyperbolic Functions

$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$\cosh^2 x = (\cosh 2x + 1)/2$$
$$\sinh^2 x = (\cosh 2x - 1)/2$$
$$\cosh^2 x - \sinh^2 = 1$$