**APPM 1360 Midterm #2 Spring 2010**

**On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor.** This exam is worth 100 points and has 5 questions.

- **Show all work!** Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes, calculators, or electronic devices are permitted.

1. (28 points) Convergence versus divergence. Explain each of your answers.

   (a) Does $\int_9^\infty e^{x^2}\,dx$ converge or diverge?

   (b) Does the sequence $a_n = n\left(1 - \cos\frac{1}{n}\right)$ for $n = 1, 2, \ldots$ converge or diverge?

   (c) Find the values of $x$, if any, for which the series $\sum_{n=1}^{\infty}(-1)^n 2^{n+1} x^n$ converges. Find the sum of the series for these values of $x$.

   (d) Does the series $\sum_{n=1}^{\infty}(1 - 1/n^2)^n$ converge or diverge?

2. (16 points) Consider the sequence given by $a_n = \dfrac{-8}{(4n-3)(4n+1)}$ for $n = 1, 2, \ldots$ and the series given by $\sum_{n=1}^{\infty} a_n$.

   (a) Does the sequence $\{a_n\}$ converge? If so, what is its limit?

   (b) Find a simple expression for the $n^{th}$ partial sum $S_n$. What is $S_{10}$?

   (c) Does the sequence $\{S_n\}$ converge? If so, what is its limit?

   (d) Does the series $\sum_{n=1}^{\infty} a_n$ converge? If so, what is its limit?

3. (14 points) Two short-answer questions. Explain your answers fully.

   (a) For what values of $p$ does the series $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^p}$ converge?

   (b) For what values of $p$ does the integral $\int_1^2 \dfrac{1}{x(\ln x)^p}\,dx$ converge?

4. (18 points) Evaluate each of the following integrals.

   (a) $\int e^{3t} \sin 2t\,dt$

   (b) $\int_0^1 \dfrac{1}{(4 - x^2)^{3/2}}\,dx$

THERE IS MORE ON THE BACK

5. (24 points) Determine whether each of the following statements is TRUE or FALSE. For this problem only, you should write TRUE or FALSE, no explanation is needed.

(a) If a sequence of positive numbers is bounded from above, then it converges.

(b) We can always write $\sum_{n=1}^{\infty}(a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$

(c) The following two series converge to the same value: $\sum_{n=0}^{\infty} \frac{1}{n!}$ and $\sum_{n=5}^{\infty} \frac{1}{(n-5)!}$.

(d) If $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty}(b_n/n)$ also converges.

(e) Consider the infinite region in the first quadrant bounded by the curve $y = 1/x$ for $x \geq 1$. The integral that gives the surface area of the solid formed by revolving this region around the x-axis is finite. Hint: Recall the formula for surface area is given by $S = \int_a^b 2\pi f(x)\sqrt{1 + (f'(x))^2}\, dx$.

(f) Consider the same region as in part (e). The integral that gives the volume of this solid is finite.

**Formulas for APPM 1360, Spring 2010, Exam 2**

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**Inverse Trigonometric Integral Identities**

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C,\ u^2 < a^2$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C,\ u^2 > a^2$$

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**Inverse Hyperbolic-Trig Integral Identities**

$$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C,\ a > 0$$
$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C,\ u > a > 0$$
$$\int \frac{du}{a^2-u^2} = = \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C,\ \text{if } u^2 < a^2$$
$$\int \frac{du}{a^2-u^2} = \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C,\ \text{if } u^2 > a^2$$
$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{u}{a}\right) + C,\ 0 < u < a$$
$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\text{csch}^{-1}\left|\frac{u}{a}\right| + C,\ u \neq 0$$

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**Identities for Hyperbolic Functions**

$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$\cosh^2 x = (\cosh 2x + 1)/2$$
$$\sinh^2 x = (\cosh 2x - 1)/2$$
$$\cosh^2 x - \sinh^2 = 1$$