APPM 1360 Midterm #3 Spring 2010

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor. This exam is worth 100 points and has 6 questions.

- **Show all work!** Answers with no justification will receive no points. **In particular, if you use the Alternating Series Test, or any other test for convergence, you must justify that your series meets the conditions of the Test.**
- Please begin each problem on a new page.
- No notes, calculators, or electronic devices are permitted.

1. (14 points) Do the following series converge absolutely, converge conditionally (but not absolutely) or diverge? Give reasons for your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2+1}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$

2. (16 points) Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^{2n}\sqrt{n^2+1}}$

   (a) Find the interval and radius of convergence. Clearly label each.

   (b) For what values of x does the series converge absolutely?

   (c) For what values of x does the series converge conditionally (but not absolutely)?

   (d) For what values of x does the series diverge?

3. (20 points) Please answer the following unrelated problems using series:

   (a) Use your knowledge of the Maclaurin series for $\sin x$ and $\cos x$ to find the first three nonzero terms of the Maclaurin series for $y = \cos x \sin(2x)$.

   (b) Find the second order Taylor polynomial, $P_2(x)$, for $f(x) = \ln(\sin x)$ at $a = \pi/4$ using the definition of the Taylor series.

4. (15 points) Evaluate $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)}{3^n}$ by following these steps:

   (a) Express $1/(1-x)$ as a geometric series.

   (b) Differentiate, with respect to x, both sides of the equation you found in part (a).

   (c) Differentiate, with respect to x, both sides of the equation you found in part (b). (This is a second differentiation.)

   (d) Multiply both sides by x^2.

   (e) Substitute in $x = 1/3$ to find the value of $\displaystyle\sum_{n=2}^{\infty} \frac{n(n-1)}{3^n}$.

THERE IS MORE ON THE BACK

5. (20 points) The function $Si(x) = \int_0^x \frac{\sin t}{t}\, dt$ is called the sine integral function. It arises in electrical engineering and physics applications.

   (a) Use your knowledge of the Maclaurin series of $\sin t$ to find a Maclaurin series for $\frac{\sin t}{t}$.

   (b) Integrate the series from part (a) to find a Maclaurin series for $Si(x)$.

   (c) Use the first two nonzero terms of the series from part (b) to estimate $Si(1)$.

   (d) Estimate the error. Explain your work.

6. (15 points) Label each of the following statements as TRUE or FALSE. If the statement is false, please correct the information in **bold**.

   (a) The **focus** of the parabola $y^2 = 10x$ is the point $(\mathbf{10}, \mathbf{0})$.

   (b) $x^2 + 4x + y^2 = 12$ is the equation of an **ellipse** with **center at** $(\mathbf{0}, \mathbf{0})$.

   (c) $5x^2 - 4y^2 = 20$ is the equation of a **hyperbola** with **asymptotes given by** $\mathbf{y} = \pm(\mathbf{5/2})\mathbf{x}$.