

**INSTRUCTIONS:** Books, notes, and electronic devices are not permitted. Write (1) **your name**, (2) **1360/EXAM 3**, (3) **instructor's name/class time** and (4) **SPRING 2011** on the front of your bluebook. Also make a scoring table with room for 6 problems and a total score. **Work all problems. Start each problem on a new page. Box your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

— SHOW ALL WORK —

1. (18 pts) Answer the following questions, justify your answers.

(a) (6 pts) Evaluate the limit using series:  $\lim_{x \rightarrow 0} \frac{\cos(x^2) - 1}{x^4}$

(b) (6 pts) Find the first four terms of the Maclaurin series of  $f(x) = \frac{x^2}{\sqrt{3+x}}$ , simplify your answer.

(c) (6 pts) Find the Maclaurin series of  $f(x) = x^3 \sin(3x)$ , give your answer in  $\sum$ -notation.

2. (12 pts) Show all work and justify your answers. Consider the series  $\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n^2}$ ,

(a) For what values of  $x$  does this series converge conditionally?

(b) For what values of  $x$  does this series converge absolutely?

(c) For what values of  $x$  does this series diverge?

3. (15 pts) Do the following series converge or diverge? Justify your answer, name any test that you use.

(a)  $\sum_{n=2}^{\infty} \frac{\ln(n)}{\ln(3n)}$

(b)  $\sum_{n=1}^{\infty} \frac{\sqrt{4n}}{n^2 - 5}$

(c)  $\sum_{n=1}^{\infty} \frac{\ln(5n)}{7n}$

4. (18 pts) Do the following series converge or diverge? If the series converges, does it converge **conditionally** or **absolutely**? Justify your answer, name any test that you use.

(a)  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{4n-5}$

(b)  $\sum_{n=1}^{\infty} (-1)^n \frac{4^n}{n!}$

(c)  $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{7n}$

5. (17 pts) Answer the following questions with as much detail as possible.

(a) (6 pts) What function does the series  $\sum_{n=0}^{\infty} \frac{3^n x^{3n+2}}{n!}$  represent?

(b) (6 pts) What number does the series  $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$  converge to?

(c) (5 pts) If we use the first four terms of the series to approximate the number in part (b), what is the error bound? Is this approximation an *underestimate* or an *overestimate*? Justify your answer.

6. (20 pts–4pts ea.) Answer “Always True” or “False” . **You do NOT need to justify your answer.**

(a) If the sequence  $\{a_n\}_{n=1}^{\infty}$  converges to zero, then the series  $\sum_{n=1}^{\infty} a_n$  will converge.

(b) If you show that  $0 \leq b_n \leq a_n$  for all  $n$ , and the series  $\sum_{n=1}^{\infty} b_n$  converges then  $\sum_{n=1}^{\infty} a_n$  converges.

(c) If  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  are both divergent series, then  $\sum_{n=1}^{\infty} a_n + b_n$  will also be divergent.

(d) If  $\sum_{n=1}^{\infty} b_n 7^n$  converges then  $\sum_{n=1}^{\infty} b_n (-5)^n$  also converges.

(e) If  $0 \leq a_n \leq b_n$  for all  $n$ , and the sequence  $\{b_n\}_{n=1}^{\infty}$  converges then the sequence  $\{a_n\}_{n=1}^{\infty}$  also converges.

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**FORMULAS ON THE OTHER SIDE.**