APPM 2350 **EXAM 2** **SUMMER 2004**

On the front of your bluebook write: (1) your name, (2) your student ID #, (3) your instructor's name (Wild or Rother), and (4) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and BOX in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and crib sheets are NOT permitted. Please start each new problem on a new page of the bluebook. Good luck and have fun!

1. (23 points) Consider a thin plate of density $\delta(x, y) = y + 1$ bounded by the curves $x = y^2$ and $x = 2y - y^2$.
   (a) Find the mass of the region.
   (b) Find the center of mass of the region.
   (c) Find the moment of inertia of the region about the x axis.

2. (32 points) Let $f(x, y) = 2x^3 - 3x^2 + y^2 - 2y + 3$.
   (a) Find all local minima, maxima, and saddle points of $f(x, y)$.
   (b) Let $g(x, y) = yf_{xx}$. Find the location of the optimal value(s) on the surface $x^2y = a$ (Report your answer in terms of the constant a).
   (c) Find the error bound on the linear approximation of $f(x, y)$ at the point (1,2) on the rectangle $0.75 \le x \le 1.25, \quad 2 - \frac{1}{12} \le y \le 2 + \frac{1}{12}$.
   (d) Find the quadratic approximation of $f(x, y)$ at the point (0,0).

3. (15 points) Consider the (ugly) integral

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{0} \frac{4\sqrt{x^2 + y^2}}{1 + (x^2 + y^2)^{3/2}} dydx.$$

   (a) Sketch the region of integration.
   (b) Reverse the order of integration. (Do *not* evaluate.)
   (c) Change the integral to polar coordinates and evaluate.

4. (15 points) Let D be the solid bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the plane $z = 1$. Set up the following integrals that give the volume of D using the given orders of integration:
   (a) $d\rho d\phi d\theta$ (spherical).
   (b) $drd\theta dz$ (cylindrical).
   (c) Find the volume of D using the triple integral of your choice.

5. (15 points) Use the substitution $u = 2x - 3y$ and $v = y - x$ to evaluate the integral $\int_{-3}^{0} \int_{x}^{x+1} 2(x - y)dydx$.

YOU WILL FIND SOME USEFUL INFORMATION ON THE BACK!

Cylindrical and spherical coordinates

Cylindrical to Rectangular	Spherical to Cylindrical	Spherical to Rectangular
$x = r\cos\theta$	$r = \rho\sin\phi$	$x = \rho\sin\phi\cos\theta$
$y = r\sin\theta$	$z = \rho\cos\phi$	$y = \rho\sin\phi\sin\theta$
$z = z$	$\theta = \theta$	$z = \rho\cos\phi$

The Second Derivative Test

Suppose $f(x, y)$ and its first and second partial derivatives are continuous in a disk centered at (a, b) and $f_x(a, b) = f_y(a, b) = 0$. Let $D = f_{xx}f_{yy} - f_{xy}^2$.

1. If $D > 0$ and $f_{xx} < 0$ at (a, b), then f has a local maximum at (a, b).
2. If $D > 0$ and $f_{xx} > 0$ at (a, b), then f has a local minimum at (a, b).
3. If $D < 0$ at (a, b), then f has a saddle point at (a, b).
4. If $D = 0$ at (a, b), then the test is inconclusive.

Mass, first moments and center of mass

Mass : $M = \iint_R \delta\, dA$

First Moments : $M_y = \iint_R x\,\delta\, dA \qquad M_x = \iint_R y\,\delta\, dA$

Center of mass : $\bar{x} = M_y/M \qquad \bar{y} = M_x/M$

Moments of Inertia : $I_y = \iint_R x^2\,\delta\, dA \qquad I_x = \iint_R y^2\,\delta\, dA \qquad I_0 = \iint_R (x^2 + y^2)\,\delta\, dA$