1. (23 points) This is homework problem 13.2.34.

   (a) (6) $M = \int_0^1 \int_{y^2}^{2y-y^2} (y+1)dxdy = \int_0^1 (2y - 2y^3)dy = \frac{1}{2}$.

   (b) (10) $M_x = \int_0^1 \int_{y^2}^{2y-y^2} y(y+1)dxdy = \int_0^1 2(y^2 - y^4)dy = \frac{4}{15}$ and $M_y = \int_0^1 \int_{y^2}^{2y-y^2} x(y+1)dxdy = \int_0^1 2(y^2 - y^4)dy = \frac{4}{15}$, hence $\bar{x} = \bar{y} = \frac{8}{15}$.

   (c) (7) $I_x = \int_0^1 \int_{y^2}^{2y-y^2} y^2(y+1)dxdy = \int_0^1 2(y^3 - y^5)dy = \frac{1}{6}$.

2. (32 points) Let $f(x,y) = 2x^3 - 3x^2 + y^2 - 2y + 3$.

   (a) (10) $f_x = 6x(x-1)$ and $f_y = 2(y-1)$ so the critical points are (0,1) and (1,1). $f_{xx} = 12x - 6$, $f_{yy} = 2$ and $f_{xy} = 0$. By the second derivative test, (0,1) is a saddle point and (1,1) is a minimum.

   (b) (10) Let $c(x,y) = x^2y - a = 0$, then $\nabla g = 12y\,\mathbf{i} + (12x-6)\,\mathbf{j}$ and $\nabla c = 2xy\,\mathbf{i} + x^2\,\mathbf{j}$. Since $x, y \neq 0$, $\nabla g = \lambda \nabla c$ gives $\lambda = \frac{6}{x}$ and $12x - 6 = 6x$. Therefore $(x,y) = (1,a)$ (is the maximum).

   (c) (7) $|f_{xx}|, |f_{xy}|, |f_{yy}| \leq 12 * 1.25 - 6 = 9$ and $|x - x_0| + |y - y_0| \leq \frac{1}{3}$ on the rectangle. Thus $|E(x,y) \leq \frac{M}{2}(|x - x_0| + |y - y_0|)^2 \leq \frac{1}{2}$.

   (d) (5) $T_2(x,y) = f(0,0) + f_x(0,0)x + f_y(0,0)y + \frac{1}{2}\left(f_{xx}(0,0)x^2 + 2f_{xy}(0,0)xy + f_{yy}(0,0)y^2\right) = 3 - 2y - 3x^2 + y^2$ (Just drop the cubic term).

3. (15 points)

   (a) (5) Lower half of a circle of radius 1.

   (b) (5) $I = \int_{-1}^0 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{4\sqrt{x^2+y^2}}{1+(x^2+y^2)^{3/2}} dxdy$

   (c) (5) $I = \int_\pi^{2\pi} \int_0^1 \frac{4r^2}{1+r^3} drd\theta = \int_\pi^{2\pi} \frac{4}{3} \ln(2)\, d\theta = \frac{4}{3}\pi \ln(2)$

4. (15 points)

   (a) $V = \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sec(\phi)} \rho^2 \sin(\phi) d\rho d\phi d\theta$.

   (b) $V = \int_0^1 \int_0^{2\pi} \int_0^z r dr d\theta dz$.

   (c) Using (b): $V = \int_0^1 \int_0^{2\pi} \frac{z^2}{2} d\theta dz = \frac{\pi}{3}$.

5. (15 points) We have $x = -u - 3v$ and $y = -u - 2v$ and hence $J(u,v) = \begin{vmatrix} -1 & -3 \\ -1 & -2 \end{vmatrix} = -1$.

   Converting the limits of integration yields:

$$\int_{-3}^0 \int_x^{x+1} 2(x-y)dydx = \int_0^1 \int_{-3v}^{-3v+3} -2v|-1|dudv = \int_0^1 -6vdv = -3.$$

   In the x-y plane, we have a parallelogram with vertices $\{(-3,-3), (-3,-2), (0,0), (0,1)\}$. In the u-v plane, we have a parallelogram with vertices $\{(-3,1), (0,1), (3,0), (0,0)\}$.