

Frenet formulas, and tangential and normal acceleration components:

$$\begin{aligned} \mathbf{T} &= \frac{\mathbf{v}}{|\mathbf{v}|} & \mathbf{N} &= \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} & \mathbf{B} &= \mathbf{T} \times \mathbf{N} \\ \kappa &= \left| \frac{d\mathbf{T}}{ds} \right| = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} & \tau &= -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N} & \frac{d\mathbf{T}}{ds} &= \kappa \mathbf{N} & \frac{d\mathbf{B}}{ds} &= -\tau \mathbf{N} \\ \mathbf{a} &= a_N \mathbf{N} + a_T \mathbf{T} & a_T &= \frac{d|\mathbf{v}|}{dt} & a_N &= \kappa |\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2} \end{aligned}$$

Arc length:

$$ds = |\mathbf{v}| dt$$

Cylindrical and spherical coordinates:

| Cylindrical to Rectangular | Spherical to Cylindrical | Spherical to Rectangular |
|----------------------------|--------------------------|----------------------------------|
| $x = r \cos \theta$ | $r = \rho \sin \phi$ | $x = \rho \sin \phi \cos \theta$ |
| $y = r \sin \theta$ | $z = \rho \cos \phi$ | $y = \rho \sin \phi \sin \theta$ |
| $z = z$ | $\theta = \theta$ | $z = \rho \cos \phi$ |

The Second Derivative Test:

Suppose $f(x, y)$ and its first and second partial derivatives are continuous in a disk centered at (a, b) and $f_x(a, b) = f_y(a, b) = 0$. Let $D = f_{xx}f_{yy} - f_{xy}^2$.

1. If $D > 0$ and $f_{xx} < 0$ at (a, b) , then f has a local maximum at (a, b) .
2. If $D > 0$ and $f_{xx} > 0$ at (a, b) , then f has a local minimum at (a, b) .
3. If $D < 0$ at (a, b) , then f has a saddle point at (a, b) .
4. If $D = 0$ at (a, b) , then the test is inconclusive.

Mass, first moments and center of mass (2-D):

$$\begin{aligned} \text{Mass} &: M = \iint_R \delta \, dA \\ \text{First Moments} &: M_y = \iint_R x \delta \, dA & M_x &= \iint_R y \delta \, dA \\ \text{Center of mass} &: \bar{x} = M_y/M & \bar{y} &= M_x/M \\ \text{Moments of Inertia} &: I_y = \iint_R x^2 \delta \, dA & I_x &= \iint_R y^2 \delta \, dA & I_0 &= \iint_R (x^2 + y^2) \delta \, dA \end{aligned}$$

Green's Theorem in a plane (The curve C is traversed counterclockwise.):

$$\begin{aligned} \text{Outward Flux} &= \oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \oint_C M \, dy - N \, dx = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) \, dA \\ \text{Circulation} &= \oint_C \mathbf{F} \cdot \mathbf{dr} = \oint_C M \, dx + N \, dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) \, dA \end{aligned}$$

Surface area of level surface $f(x, y, z) = c$: $S = \iint_S d\sigma$ where $d\sigma = \frac{|\nabla f|}{|\nabla f \cdot \mathbf{p}|} dA$