

Projections and distances

$$\text{proj}_{\mathbf{A}} \mathbf{B} = \left( \frac{\mathbf{A} \cdot \mathbf{B}}{\mathbf{A} \cdot \mathbf{A}} \right) \mathbf{A} \quad d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|} \quad d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$

Arc length, Frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}| dt \quad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \quad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \quad \mathbf{B} = \mathbf{T} \times \mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa \mathbf{N} \quad \frac{d\mathbf{B}}{ds} = -\tau \mathbf{N} \quad \kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}$$

$$\tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N} = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \ddot{\ddot{x}} & \ddot{\ddot{y}} & \ddot{\ddot{z}} \end{vmatrix}}{|\mathbf{v} \times \mathbf{a}|^2}$$

$$\mathbf{a} = a_N \mathbf{N} + a_T \mathbf{T} \quad a_T = \frac{d|\mathbf{v}|}{dt} \quad a_N = \kappa |\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$

Directional derivative, Discriminant Lagrange Multipliers, and Estimating Change

$$\frac{df}{ds} = (\nabla f) \cdot \mathbf{u} = D_{\mathbf{u}} f \quad f_{xx}f_{yy} - (f_{xy})^2 \quad \nabla f = \lambda \nabla g \quad df = (\nabla f|_{P_0} \cdot \mathbf{u}) ds$$

Linear Approximation Error,  $E(x, y)$

$$|E(x, y)| \leq \frac{1}{2} M (|x - x_0| + |y - y_0|)^2, \quad \text{where } \max\{|f_{xx}|, |f_{xy}|, |f_{yy}|\} \leq M$$

Polar coordinates  $x = r \cos \theta \quad y = r \sin \theta \quad r^2 = x^2 + y^2 \quad dA = dx dy = r dr d\theta$

Cylindrical and spherical coordinates

Cylindrical to Rectangular	Spherical to Cylindrical	Spherical to Rectangular
$x = r \cos \theta$	$r = \rho \sin \phi$	$x = \rho \sin \phi \cos \theta$
$y = r \sin \theta$	$z = \rho \cos \phi$	$y = \rho \sin \phi \sin \theta$
$z = z$	$\theta = \theta$	$z = \rho \cos \phi$

$$dV = dx dy dz = r dz dr d\theta = \rho^2 \sin \phi d\rho d\phi d\theta$$

Substitutions in multiple integrals

$$\iint_R f(x, y) dx dy = \iint_G f(x(u, v), y(u, v)) |J(u, v)| du dv \quad \text{where } J(u, v) = \frac{\partial x}{\partial u} \frac{\partial y}{\partial v} - \frac{\partial y}{\partial u} \frac{\partial x}{\partial v}$$

Mass, first moments and center of mass

$$\begin{aligned} \text{Mass} \quad M &= \iiint_R \delta dV \\ \text{Moments} \quad M_{yz} &= \iiint_R x \delta dV \quad M_{xz} = \iiint_R y \delta dV \quad M_{xy} = \iiint_R z \delta dV \\ \text{Center of mass} \quad \bar{x} &= M_{yz}/M \quad \bar{y} = M_{xz}/M \quad \bar{z} = M_{xy}/M \end{aligned}$$