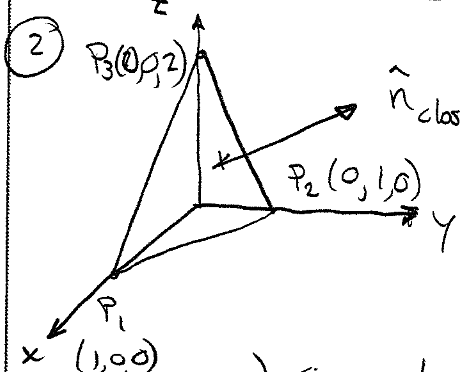
1.
a) Not always true
b) always true
c) always true
d) not always true

5.
a) 4, 7
b) 3
c) 1
d) 5

2.



Define vectors $\underline{P_{12}} = -\hat{\imath} + \hat{\jmath}$

$\underline{P_{13}} = -\hat{\imath} + 2\hat{k}$

Then $\underline{n}_{closed} = \underline{P_{12}} \times \underline{P_{13}} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$

a) Given that (non-unit) normal to the closed plane is $\underline{n}_{closed} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$, then the std. eqn. of the plane must be $2x + 2y + z = D$.

Evaluating this at P_1 gives $D = 2$.

So $\hat{n}_{closed} = \dfrac{2\hat{\imath} + 2\hat{\jmath} + \hat{k}}{3}$ and $2x + 2y + z = 2$ ←

b) Note that a normal to the closed position would be

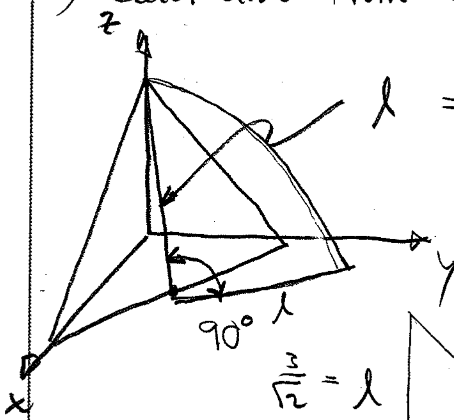
$\underline{n}_{open} = \underline{n}_{clos.} \times \underline{P_{12}} = \dots = \hat{\imath} + \hat{\jmath} - 4\hat{k}$

Thus the open plane must be of the form

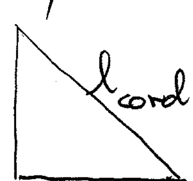
$x + y - 4z = d$. Eval. at P_1 to get $d = 1$.

Thus $\hat{n}_{open} = \dfrac{\hat{\imath} + \hat{\jmath} - 4\hat{k}}{\sqrt{18}}$ $\quad x + y - 4z = 1$ ←

c) Calc. dist from P_3 to line through P_1 + P_2



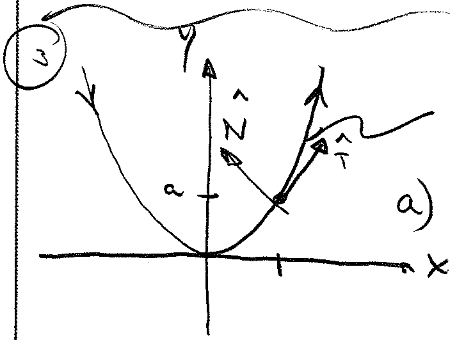
$$l = \frac{|\underline{P_{13}} \times \underline{P_{12}}|}{|\underline{P_{12}}|} = \frac{|-\underline{n}_c|}{|\underline{P_{12}}|} = 3/\sqrt{2}$$



$l_{cord} = \sqrt{l^2 + l^2} = 3$ ←

2 d) $\text{area}_{door} = \dfrac{|\underline{P}_{12} \times \underline{P}_{13}|}{2} = \dfrac{|\underline{n}_c|}{2} = 3/2$

3



path is on curve $y = f(x) = ax^2$

a) $k = \dfrac{|f''|}{\left[1+(f')^2\right]^{3/2}} = \dfrac{2a}{\left(1+(2ax)^2\right)^{3/2}}$

b) $k\Big|_{(1,a)} = \dfrac{2a}{\left(1+(2a)^2\right)^{3/2}}$

c) k is max. when the denom. of k is minimized, ie when $(1+4a^2x^2)$ is minimized at $x=0$, $k = 2a$

d) k decreases as $x \to \infty$. In fact $\lim\limits_{x \to \pm\infty} k = 0$

but k actually has no minimum.

e) $\underline{a} = a_T \hat{T} + a_N \hat{N}$ where $a_T = \dfrac{d|\underline{v}|}{dt} = 0$

and $a_N = k|\underline{v}|^2 = \dfrac{2a\,|\underline{v}|^2}{(1+4a^2x^2)^{3/2}}$

so $\underline{a}\Big|_{(1,a)} = \dfrac{2a|\underline{v}|^2}{(1+4a^2)^{3/2}} \hat{N}$ but we still need $\hat{N}\Big|_{(1,a)}$

Since $f'(x) = 2ax$, then $\underline{T}\Big|_{(1,a)} = \hat{\imath} + 2a\hat{\jmath}$ and $\underline{N} = -2a\hat{\imath} + \hat{\jmath}$ (non-unit versions)

so $\hat{N}\Big|_{(1,a)} = \dfrac{-2a\hat{\imath} + \hat{\jmath}}{\sqrt{1+4a^2}}$ Hence $\underline{a}\Big|_{(1,a)} = \dfrac{2a|\underline{v}|^2}{(1+4a^2)^{3/2}} \dfrac{(-2a\hat{\imath}+\hat{\jmath})}{(1+4a^2)^{1/2}}$

$= \dfrac{2a|\underline{v}|^2}{(1+4a^2)^2}(-2a\hat{\imath}+\hat{\jmath})$ + $|\underline{v}| = 5\,\text{m/s}$

f) Torsion of a path in a plane is $\tau = 0$.

4

$\underline{r}(t) = \frac{t}{\sqrt{2}}\hat{i} + \frac{t}{\sqrt{2}}\hat{j} + \left(1 - \frac{t^2}{2}\right)\hat{k}$

$\underline{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + (-t)\hat{k}$ + $|\underline{v}| = \sqrt{1+t^2}$

a) Joey hits ground when $\hat{k}$ component of $\underline{r}$ is 0.
So $1 - \frac{t^2}{2} = 0$ gives $t = \sqrt{2}$ $\longleftarrow$

b) Arc length $s = \int_{t=0}^{\sqrt{2}} |\underline{v}|\,dt = \int_{t=0}^{\sqrt{2}} \sqrt{1+t^2}\,dt$

c) $= \left[\frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\ln\left(t + \sqrt{1+t^2}\right)\right]_{t=0}^{\sqrt{2}}$

$= \frac{\sqrt{6}}{2} + \frac{\ln\left(\sqrt{2}+\sqrt{3}\right)}{2}$ $\longleftarrow$