

### ORALS QUESTIONS FOR CALCULUS III

1. How do you know if two vectors are equal?
2. Do two equal vectors have to have the same initial point?
3. How do I know that two vectors are parallel? Give an example of two parallel vectors.
4. How do I indicate the direction of the vector  $\langle 1, 2 \rangle$ ?
5. How do you add two vectors? (component wise) Draw a picture of the sum of A and B in #3. What does vector addition have to do with a parallelogram.
5. How do you find the vector between two points? Is there only one? How are the two related?
6. Suppose you have the function  $y = 3x^2 + 4$ . Draw the graph and find (1,7). What would a vector tangent to the curve at (1,7) look like? How many vectors are there that are tangent to the curve at that point? Infinitely many. Find one such vector.
7. How would you find a vector that is normal to  $y = 3x^2 + 4$  at (1,7)? Is there only one normal vector? One unit normal vector?
8. In 3-dimensional space we divide space into \_\_\_ octants. Where is the first octant?
9. Name three types of vector multiplication and explain the result in each case.
10. Give the definition of dot and cross product:
11. What would the graph of the following look like?  $x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$ . Use completing the square to put the equation in standard form.
12. How do you find the midpoint of a line segment?
13. How do you find the angle between two vectors: eg.  $i - 2j - 2k$  and  $6i + 2j + 3k$
14. Which of the following are true? If not always true then give a counter example.
  - a)  $A \cdot B = B \cdot A$
  - b)  $A \cdot (B + C) = A \cdot B + A \cdot C$
  - c)  $A \cdot (B \times C) = A \cdot B \times A \cdot C$
  - d)  $|A \times B| = |B \times A|$
  - e)  $A \times B = B \times A$

- f)  $(A \times B) \cdot A = 0$
- g) The linear equation  $Ax + By + Cz = D$  represents a line in three dimensions.
- h)  $(A \times B) \cdot A = B \cdot (A \times B)$
- i) If  $(A \cdot B) = 0$ , then  $A = 0$  or  $B = 0$ .
- j.  $(A + B) \times B = A \times B$
- k. The cross product of two unit vectors is a unit vector.
- m. The set of points  $k(A \{ (x, y, z) | x^2 + y^2 = 4 \})$  is a circle
- n. If  $f(x, y, z)$  is continuous at  $(1, 0, -3)$  then it is differentiable there.
15. Draw two vectors  $\mathbf{u}$  and  $\mathbf{w}$ , and show where the projection of  $\mathbf{u}$  onto  $\mathbf{w}$  would be, and label that vector  $\mathbf{v}$ . How do you find the vector  $\mathbf{v}$ ?
16. What is true of the cross product of  $\mathbf{A}$  and  $\mathbf{B}$  if the vectors are parallel? When the dot product is zero, what does that tell you?
17. Why are you interested in whether a curve is smooth or not?
18. What values of "a" would make the following vectors orthogonal.  $\langle 3, 2, a \rangle$  and  $\langle 2a, 4, a \rangle$ ?
19. Find the following equations:
- the line through  $(4, -1, 2)$  and  $(1, 1, 5)$
  - the line through  $(1, 0, -1)$  and parallel to the line  $\frac{1}{3}(x - 4) = \frac{1}{2}y = z + 2$
  - the line through  $(-2, 2, 4)$  and perpendicular to the plane  $2x - y + 5z = 12$ .
  - The plane through  $(2, 1, 0)$  and parallel to  $x + 4y - 3z = 1$
  - The plane through  $(3, -1, 1)$ ,  $(4, 0, 2)$  and  $(6, 3, 1)$
20. Find the distance from the origin to the line  $x = 1 + t$ ,  $y = 2 - t$ ,  $z = -1 + 2t$ .
21. Where does the line  $x = 2 - t$ ,  $y = 1 + 3t$ ,  $z = 4t$  intersect the plane  $2x - y + z = 2$
22. Are the planes  $x + y - z = 1$  and  $2x - 3y + 4z = 5$  parallel? Perpendicular? Neither?
23. What is the angle between the two planes in 22?
24. Find the points where the line  $x = 1 + 2t$ ,  $y = -1 - t$ ,  $z = 3t$  intersects the three coordinate planes.
25. Identify and sketch the following quadric surfaces:
- $x^2 + (y - 3)^2 + z^2 = 4$

- b)  $4x^2 + 9y^2 + z^2 = 36$
- c)  $4x^2 - 9y^2 - z^2 = 36$
- d)  $-4x^2 + 9y^2 + z^2 = 36$
- e)  $f_x^r + f_y^r = z^r$
- f)  $f_x^r - f_y^r = z$
- g)  $-f_x^r - f_y^r = z$