

INSTRUCTIONS: Electronic devices, books, and crib sheets are not permitted. Write your (1) name, (2) instructor's name, and (3) recitation number on the front of your bluebook. Work all problems. Show your work clearly. Note that a correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

Projections and distances

$$\text{proj}_{\mathbf{A}} \mathbf{B} = \left(\frac{\mathbf{A} \cdot \mathbf{B}}{\mathbf{A} \cdot \mathbf{A}} \right) \mathbf{A} \quad d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|} \quad d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$

Arc length, frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}| dt \quad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \quad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \quad \mathbf{B} = \mathbf{T} \times \mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa \mathbf{N} \quad \frac{d\mathbf{B}}{ds} = -\tau \mathbf{N} \quad \kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{|1 + (f'(x))^2|^{3/2}} = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{|\dot{x}^2 + \dot{y}^2|^{3/2}} \quad \tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N}$$

$$\mathbf{a} = a_N \mathbf{N} + a_T \mathbf{T} \quad a_T = \frac{d|\mathbf{v}|}{dt} \quad a_N = \kappa |\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$

Directional derivative, discriminant, and Lagrange multipliers

$$\frac{df}{ds} = (\nabla f) \cdot \mathbf{u} \quad f_{xx}f_{yy} - (f_{xy})^2 \quad \nabla f = \lambda \nabla g, \quad g = 0$$

Taylor's formula (at the point (x_0, y_0))

$$f(x, y) = f(x_0, y_0) + \left[(x - x_0)f_x(x_0, y_0) + (y - y_0)f_y(x_0, y_0) \right]$$

$$+ \frac{1}{2!} \left[(x - x_0)^2 f_{xx}(x_0, y_0) + 2(x - x_0)(y - y_0)f_{xy}(x_0, y_0) + (y - y_0)^2 f_{yy}(x_0, y_0) \right]$$

$$+ \frac{1}{3!} \left[(x - x_0)^3 f_{xxx}(x_0, y_0) + 3(x - x_0)^2(y - y_0)f_{xxy}(x_0, y_0) \right.$$

$$\left. + 3(x - x_0)(y - y_0)^2 f_{xyy}(x_0, y_0) + (y - y_0)^3 f_{yyy}(x_0, y_0) \right] + \dots$$

Linear approximation error

$$|E(x, y)| \leq \frac{1}{2} M (|x - x_0| + |y - y_0|)^2, \quad \text{where } \max\{|f_{xx}|, |f_{xy}|, |f_{yy}|\} \leq M$$