Orals Questions – Test 2 – Calculus III

1. True or False:

   a. If $f(x,y) \to L$ as $(x,y) \to (a,b)$ along every straight line through (a,b), then the limit as $(x,y) \to (a,b)$ of f(x,y) = L (*false – it must be along all paths*)
   b. If $f_x(a,b)$ and $f_y(a,b)$ both exist, then f is differentiable at (a,b) (*false, they must be continuous at (a,b)*
   c. If f has a local min at (a,b) and f is differentiable at (a,b), then $\nabla f(a,b) = 0$. *True*
   d. If f is a function, then $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$ (*False – only if it is continuous at (a,b))*
   e. If f(x,y) = lny, then $\nabla f = \frac{1}{y}$ (*False – the gradient is a vector <0,1/y>)*
   f. If (2,1) is a critical point and $f_{xx}(2,1) f_{yy}(2,1) < [f_{xy}(2,1)]^2$, then f has a saddle point at (2,1). (*True – the discriminant is less than ze*ro)
   g. If $f_y(a,b)$ exists, then it could be a number or a function. *(False – you are evaluating $f_y(a,b)$ at (a,b) so it must be a number.*
   h. $f_{xy}(2,1) = f_{yx}(2,1)$ if both are continuous on D which contains (2,1) *(True)*

1. What does $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$ mean? How could you show that such a limit does not exist?
2. What does it mean to say that f(x,y) is continuous at (a,b) mean?
3. How do you interpret $f_x(a,b)$ and $f_y(a,b)$ geometrically?
4. What do you need to know in order to find the equation of a plane? Draw a graph to show why the normal and a point in the plane are enough.
5. What would the linearization of f at (a,b) look like? (*a tangent plane to the surface at (a,b))*
6. If z = f(x,y), then what are the differentials dx, dy and dz at (a,b)?
   dz = $f_x(a,b)$ (x-a) + $f_y(a,b)$ (y-b)
   in general dz = $f_x(x,y)dx + f_y(x,y)dy$
7. Write an expression for the directional derivative of f at (a,b) in the direction of the vector <c,d>. How do you interpret this derivative as a rate? How do you interpret it geometrically?
8. Define the gradient vector for a function of two variables. What does the magnitude of the gradient vector tell you?
9. Geometrically, what does it mean to say that there is a saddle point at (a,b)? How would you determine that there is a saddle point at (a,b)?
10. If f has a local max at (a,b), what can you said about the first partial derivatives at (a,b)?

11. Explain how the method of LaGrange Multipliers works in finding extreme values of f(x,y,z) subject to the constraint g(x,y,z) = k.
12. Find and sketch the domain for the following functions:
    a. f(x,y) = ln(x + y + 1)
    b. f(x,y) = $\sqrt{4-x^2-y^2}$
13. Sketch several level curves for f(x,y) = $\sqrt{4x^2+y^2}$
14. Evaluate the following limits or show that they do not exist

    a. $\lim_{(x,y)\to(1,1)} \frac{2xy}{x^2+2y^2}$ b. $\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}$
15. Find the second partials of v = r cos(s + 2t)

16. Find the first partial derivatives:
    a. f(x,y) = ln(x + y + 1)
    b. u = $\sqrt{4x^2+y^2}$
    c. T(p,q,r) = p ln(q + e^t)
17. Find the equation of the tangent plane and the normal line for:
    a. z = $3x^2 - y^2 + 2x$ (1,-2,1)
    b. sin(xyz) = x +2y + 3z (2,-1,0)
18. Find points on the hyperboloid $x^2 + 4y^2 - z^2 = 4$ where the tangent plane is parallel to the plane 2x + 2y + z = 5
19. Find du if u = ln (1+s e^{2t})
20. Find the linear approximation of the function f(x,y,z) = $x^3\sqrt{z^2+y^2}$ at the point (2,3,4) and use it to estimate the number $(1.98)^3\sqrt{(3.97)^2+(3.01)^2}$
21. The two legs of a right triangle are measured as 5 m and 12 m with a possible error in measurement of at most .2cm in each. Use differentials to estimate the maximum error in the calculated value of the area of the triangle.
22. Use L.M. to solve: A package in the shape of a rectangular box can be mailed by the US Postal office if the sum of its length and girth (the perimeter of a cross section perpendicular to the length) is at most 108 in. Find the dimensions of the package with the largest volume that can be mailed.
23. Find the absolute max and min of f(x,y) = $4xy^2 - x^2y^2 - xy^3$ where d is the closed triangular region in the xy plane with vertices (0,0), (0,6) and (6,0).
24. Find the extreme values of the function f(x,y) = $x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$
25. Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from (3,1,-1)