Date: 14^{th} *January, 2010*

1. Read section **10.1** , **10.2** and **10.3** from your textbook.
2. **Definition *Vector***: It is a mathematical object characterized by (1) magnitude (or size) and (2) direction.
   Note: It does not matter where the object is located in space. This means one can think of 2 spatially separated objects which have the same magnitude and direction and hence both can be represented by the same vector. This should become further clear from figure 1 below.

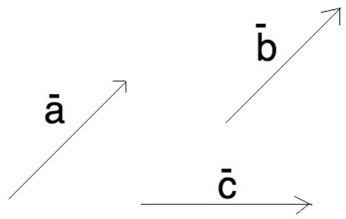

Figure 1: vectors

Here you must convince yourself that $\vec{a} = \vec{b} \neq \vec{c}$ assuming that the length of each of the objects is the same.
Now recall the **mid point formula**:- Let O be the reference point, A and B be any other points in space then the midpoint, C of the line segment AB is such that $\vec{OC} = \frac{\vec{OA}+\vec{OB}}{2}$.

Sample problems: (use properties of vectors to answer the following)

(a) Given: Parallelogram ABCD. Prove that the diagonals of the parallelogram bisect each other.

(b) Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

3. The concept of dot product (a.k.a. scalar product, inner product) will be introduced to you in class tomorrow. Try the following exercise problems over the weekend based on what you learn in class tomorrow. We will briefly talk about these problems next week.

Optional fun exercises:

(a) For any vectors $\vec{u}$ and $\vec{v}$, show that $|\vec{u}.\vec{v}| \leq |\vec{u}||\vec{v}|$
(b) For any vectors $\vec{u}$ and $\vec{v}$, show that $|\vec{u}+\vec{v}| \leq |\vec{u}| + |\vec{v}|$

Sketch of solutions to the sample problems:

a) First, draw a parallelogram ABCD. Draw the diagonals. All you need to show is that if a point (say X) is the midpoint of the line segment AC; then $\vec{BX} = \vec{XD}$.

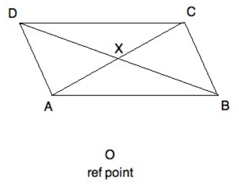

Figure 2: the parallelogram

Now, $\vec{OX} = \frac{\vec{OA}+\vec{OC}}{2}$. So, $\vec{BX} = \vec{OX} - \vec{OB} = \frac{\vec{OA}+\vec{OC}}{2} - \vec{OB} = \frac{\vec{OA}+\vec{OC}-2\vec{OB}}{2} = \frac{-\vec{AB}+\vec{BC}}{2}$. Similarly, $\vec{XD} = \vec{OD} - \vec{OX} = ... = \frac{\vec{AD}-\vec{DC}}{2} = \frac{\vec{BC}-\vec{AB}}{2}$, where the last equality was obtained by verifying from the figure 2. Hence we showed $\vec{BX} = \vec{XD}$.

b) This problem is actually easier than the one above ! Again, as always, you should begin by drawing the triangle ABC and then mark the mid points M and L on AB and AC respectively.
Then proceed by arguing that $\vec{ML} = \vec{OL} - \vec{OM} = \frac{\vec{OA}+\vec{OC}}{2} - \frac{\vec{OA}+\vec{OB}}{2} = \frac{\vec{OC}-\vec{OB}}{2} = \frac{\vec{BC}}{2}$.
Now the result should follow immediately !

Have a fantabulous weekend ! ☻