APPM 2350 **EXAM #3** **Summer 2011**

On the front of your bluebook write: (1) your name, (2) your student ID number, (3) your instructor's name, and (4) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and BOX in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and crib sheets are NOT permitted. Please start each new problem on a new page of the bluebook.

1. Consider the bottom half ($y \leq 0$) of a disc of radius $1m$. The half disc has a constant mass density of $3kg/m^2$.
   (a) What is the total mass of the disc?
   (b) Where is the center of mass of the disc?
   (c) What is the moment of inertia of the half disc around the x axis?
   (d) What is the moment of inertia of the half disc around the y axis?

2. Consider the region bounded by the curves $y = \frac{1}{x}$, $y = \frac{3}{x}$, $y = \sqrt{x}$, $y = 2\sqrt{x}$ in the first quadrant.
   (a) Sketch the region in the xy-plane (You must label all curves).
   (b) Setup an integral in xy-space that represents the area of the region (use the order dydx).
   (c) Use the transformations $u = xy$ and $v = \frac{y^2}{x}$ to transform the region and sketch it in uv-space (You must label all curves).
   (d) Setup an integral in uv-space that represents the area of the original region in xy space (use the order dvdu).
   (e) Evaluate one of the integrals.

3. Consider the integral $\int_0^{2\pi} \int_0^{\sqrt{3}} \int_{\frac{r}{\sqrt{3}}}^{1} r\,dz\,dr\,d\theta$.
   (a) Sketch the region in the $r - z$ plane and label all bounding curves. (You can buy this sketch for 5pts)
   (b) Express this integral in $drdzd\theta$ order.
   (c) Write the integral to compute the volume of this region in spherical coordinates using the order $d\rho d\phi d\theta$.
   (d) Write the integral to compute the volume of this region in spherical coordinates using the order $d\phi d\rho d\theta$.
   (e) Evaluate one of the integrals.

Projections and distances $\quad \text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \quad d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|} \quad d = \left|\overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|}\right|$

Arc length, frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}|\,dt \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N} \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N} \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{|\,1+(f'(x))^2\,|^{3/2}} = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{|\dot{x}^2+\dot{y}^2|^{3/2}} \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T} \qquad a_T = \frac{d|\mathbf{v}|}{dt} \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$

Directional derivative, discriminant, and Lagrange multipliers

$$\frac{df}{ds} = (\nabla f) \cdot \mathbf{u} \qquad f_{xx}f_{yy} - (f_{xy})^2 \qquad \nabla f = \lambda \nabla g, \quad g = 0$$

Polar coordinates $\quad x = r\cos\theta \quad y = r\sin\theta \quad r^2 = x^2 + y^2 \quad dA = dx\,dy = r\,dr\,d\theta$

Cylindrical and spherical coordinates

Cylindrical to Rectangular	Spherical to Cylindrical	Spherical to Rectangular
$x = r\cos\theta$	$r = \rho\sin\phi$	$x = \rho\sin\phi\cos\theta$
$y = r\sin\theta$	$z = \rho\cos\phi$	$y = \rho\sin\phi\sin\theta$
$z = z$	$\theta = \theta$	$z = \rho\cos\phi$

$$dV = dx\,dy\,dz = r dz\,d\theta\,dr = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$$

Substitutions in multiple integrals

$$\iint_R f(x,y)\,dx\,dy = \iint_G f\left(x(u,v), y(u,v)\right)\,|J(u,v)|\;du\,dv \quad \text{where} \quad J(u,v) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}$$

Mass, moments, and center of mass $\quad$ Mass $\quad M = \iint_R \delta\,dA$

Moments $\quad M_x = \iint_R y\,\delta\,dA \qquad M_y = \iint_R x\,\delta\,dA \qquad$ Center of mass $\quad \bar{x} = M_y/M \qquad \bar{y} = M_x/M$