

INSTRUCTIONS: Computers, calculators, books are not permitted. A one page $8\frac{1}{2} \times 11$ inch hand-written sheet of notes is permitted. Write your (1) name, (2) instructor's name, and (3) lecture number (010 or 020) on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (20 points) Mark the following statements as "True" or "False." Note that for a statement to be true, it must be *always* true. Here \mathbf{A} , \mathbf{B} and \mathbf{C} are nonzero vectors (thus $\mathbf{A} \neq \mathbf{0}$, etc.), and $\mathbf{r}(t)$ is a differentiable vector function of t . No work need be shown for this problem.
 - (a) $\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}$ is a unit vector.
 - (b) $\frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A}||\mathbf{B}|}$ is a vector perpendicular to \mathbf{A} .
 - (c) $\mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A} = \mathbf{0}$.
 - (d) $\mathbf{A} \times (\mathbf{B} \cdot \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$.
 - (e) If $\frac{d}{dt}|\mathbf{r}(t)| = 0$ for all t , then the curve defined by $\mathbf{r}(t)$ is a straight line.
 - (f) $\mathbf{A} \cdot (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = 0$ defines a plane containing the origin.
2. (20 points) Let P be the plane $2x + y - z = 3$, and L be the line $\mathbf{r}(t) = t\mathbf{i} + (1 - t)\mathbf{j} + 3t\mathbf{k}$.
 - (a) Find a unit vector normal to P .
 - (b) Find a unit vector normal to L .
 - (c) Find a point of intersection of P and L .
 - (d) Find the distance from the point $\mathbf{r}(1)$ on L to the plane P .
3. (20 points) Using sensors mounted aboard a spacecraft, it was determined that the velocity of the spacecraft was $\mathbf{v}(t) = t\mathbf{i} + 2t^{3/2}\mathbf{j} + 2t^2\mathbf{k}$, for $t \geq 0$. For $t \geq 0$, determine the following:
 - (a) the speed of the spacecraft
 - (b) the vector $\frac{d\mathbf{v}}{ds}$; Note: you do not actually need to find the arc length s to do this calculation.
 - (c) the distance traveled by the spacecraft from time $t = 0$ to $t = T$; Hint: the integrand involves a perfect square.
 - (d) the position of the spacecraft, if the initial position of the craft at time $t = 0$ was

$$\mathbf{r}_0 = \frac{1}{2}\mathbf{i} + \frac{4}{5}\mathbf{j} + \frac{2}{3}\mathbf{k}$$
4. (20 points) Consider a coordinate system with $z = 0$ (the x - y plane) representing the ground, and the z -axis pointing up. Your path on an amusement park ride, relative to the coordinate system, can be described by $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + (1 - \cos t)\mathbf{k}$, for $t \geq 0$. Note that at time $t = 0$ you are on the ground.
 - (a) Find $\mathbf{v}(t)$.
 - (b) Find your maximum and minimum speed.
 - (c) As you ride, you look straight ahead in the "forward direction." That is, you constantly look in the direction of the tangent to your path. At time $t = 3\pi/2$, what are the coordinates of the point on the ground at which you are looking?

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5. (20 points) Match each of the pictures shown (a)-(f) with one of the equations (1)-(9). Note there are more equations than pictures, so some equations will be unused. No work need be shown for this problem.

- (1) $3x^2 - y^2 + z^2 = 2$ (2) $x^2 - 3y^2 - z^2 = 2$ (3) $-x^2 - 3y^2 + z^2 = 2$
 (4) $-3x - y + z = 2$ (5) $3x - y^2 + z^2 = 2$ (6) $3x^2 - y^2 + z = 2$
 (7) $3x^2 + z = 2$ (8) $3y^2 + z = 2$ (9) $x + 3y = 2$ and $x + z = 2$

