INSTRUCTIONS: Computers, calculators, books are not permitted. A one page $8\frac{1}{2} \times 11$ inch hand-written sheet of notes is permitted. Write your (1) name, (2) instructor's name, and (3) lecture number (010 or 020) on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and $\boxed{\text{box}}$ your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

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1. (20 points) Evaluate each of these statements as either always TRUE or FALSE. No work need be shown for this question. You must answer with the full word "TRUE" or "FALSE" — answers using only "T" or "F" **will not be graded**.

   (a) If a function $f(x, y)$ is twice differentiable in a region surrounding point $P(a, b)$ and has a local maximum at (a, b), then $f_{yy}\Big|_P < 0$.

   (b) If a function $f(x, y, z)$ has a critical point at $P(a, b, c)$, then $\nabla f\Big|_P = 0$.

   (c) The vector $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ is normal to the surface $z^2 + y^2 = \dfrac{x^2}{2} + yz$ at the point $(x, y, z) = (2, 0, 1)$.

   (d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2} = 0.$

   (e) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2} = 0.$

   (f) For a curve $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, the unit binormal vector $\mathbf{B}$ is either $\mathbf{k}$ or $-\mathbf{k}$.

2. (20 Points) A particle moves along the curve $\mathbf{r}(t) = t^2\mathbf{i} + \left(t + \dfrac{t^3}{3}\right)\mathbf{j} + \left(t - \dfrac{t^3}{3}\right)\mathbf{k}$.

   (a) Find the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of the particle.

   (b) Find the unit tangent vector $\mathbf{T}$.

   (c) At what value(s) of t is the particle moving horizontally?

   (d) Compute the particle's tangential and normal accelerations a_T and a_N. (Hint: you do not need to find the principle unit normal $\mathbf{N}$).

3. (20 Points) You are asked to construct a grain silo of volume 1000 m^3 from a cylinder of radius r and height h, topped by a hemisphere of radius r. The cost to build the silo is \$1/m^2 for the sidewalls, and \$3/m^2 for the hemispherical top. (Hint: the volume of a sphere is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$.)

   (a) What is the function describing the total volume of the silo, V, in terms of r and h?

   (b) What is the function describing the total cost of the silo, C, in terms of r and h?

   (c) Your task is to build the silo for minimum cost. Set up the problem using Lagrange multipliers.

   (d) Using your result from part (c), find the values of r and h that give the silo of minimum cost.

4. (20 Points) An open cockpit plane is flying in a circular path about the airport while waiting for clearance to land. Measured from the airport, its path is described by $\mathbf{r}(t) = 10\cos(2t)\mathbf{i} + 10\sin(2t)\mathbf{j} + 10\mathbf{k}$ where distances are measured in kilometers and time is measured in minutes. The temperature in degrees C, as a function of position, is given by $T(x, y, z) = 0.3xy + 0.5z$. Answer the following questions assuming the plane is located at $(0, 10, 10)$.

   (a) What rate of temperature change does the pilot experience in °C/min?

   (b) In °C/km?

   (c) If the pilot suddenly turned and flew toward the airport while maintaining its altitude, by approximately how many degrees C would the temperature change after the pilot travels 0.1 km in the new direction?

5. (20 Points) You have just filed a mining claim on some property near Gold Hill. The boundaries of your claim are described by the lines $x = 0$, $y = 3$ and $y = x$ in the first quadrant. You don't know it, but the density of gold (in micro grams per cubic centimeter) on your claim is described by $f(x, y) = x^2 - 4x + y^2 - 2y + 5$. At what locations on your claim would your mining efforts be the most, and least, productive?