

**INSTRUCTIONS:** Books and electronic devices are not permitted. A one-page  $8\frac{1}{2} \times 11$  inch handwritten sheet of notes is permitted. Write your (1) name, (2) instructor's name, and (3) lecture number (010 or 020) on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

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1. (20 points) Use the Taylor formula to find the quadratic approximation of the function  $f(x, y) = e^y \cos x$  near the origin.
2. (20 points) Integrate the function  $f(x, y, z) = xy + y + z$  along the curve defined by  $\mathbf{r}(t) = 2t \mathbf{i} + t \mathbf{j} + (2 - 2t) \mathbf{k}$  for  $0 \leq t \leq 1$ .
3. (20 points) Consider the integral  $I = \iint_R \cos(xy) \, dx \, dy$  where  $R$  is the region in the first quadrant bounded by the curves  $xy = \pi/2$ ,  $xy = \pi$ ,  $y = e^2x$  and  $y = e^4x$ .
  - (a) Use the transformation  $x = u/v$ ,  $y = uv$  with  $u > 0$  and  $v > 0$  to rewrite the integral over an appropriate region  $R'$  in the  $uv$ -plane.
  - (b) Make two clear sketches, one of the original region of integration  $R$  in the  $xy$ -plane, and one of the region  $R'$  in the  $uv$ -plane. (Be sure to label your axes, boundaries, and so on!)
  - (c) Evaluate the integral in terms of  $uv$  over the region  $R'$ .
4. (20 points) Consider the volume of the region in the first octant bounded by the planes  $x = 0$ ,  $y = 0$ ,  $z = 0$  and  $x + y + z = 1$ .
  - (a) Clearly sketch the region of integration in the  $xyz$ -coordinate system.
  - (b) Set up the integral to calculate the volume  $V$  in cartesian coordinates in the order  $dx \, dy \, dz$ .
  - (c) Set up the integral in spherical coordinates in the order  $d\rho \, d\phi \, d\theta$ .
  - (d) Evaluate **one** of the integrals to find the volume of the region.
5. (20 points) Consider the volume of the object bounded above by the surface  $z = 2 - \sqrt{x^2 + y^2}$  and below by the surface  $z = \sqrt{x^2 + y^2}$ .
  - (a) Make a clear sketch the object in the  $xyz$ -coordinate system.
  - (b) Set up the integral to calculate the volume  $V$  in cylindrical coordinates in the order  $dz \, dr \, d\theta$ .
  - (c) Set up the integral in cylindrical coordinates in the order  $dr \, dz \, d\theta$ .
  - (d) Set up the integral in spherical coordinates in the order  $d\rho \, d\phi \, d\theta$ .
  - (e) Evaluate **one** of the integrals to find the volume of the object.