
INSTRUCTIONS: Computers, calculators, books, notes, flying monkeys, *etc.* are not permitted. Some (possibly) useful formulae are attached. Write your name, your instructor's name, and the color of your exam sheet on the front of your bluebook. Work all problems. Start each problem on a **new page**. **Show your work clearly, label your axes, and $\boxed{\text{box}}$ your final answer**. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

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1. (20 points) Consider the function $f(x, y) = 4xy$ on the closed region $R = \{(x, y) : x^2+y^2 \leq 8\}$
   (a) Sketch R, label the intercepts.
   (b) Find all absolute extrema of $f(x, y)$ on R.
   (c) On your sketch of R, show the locations of the absolute extrema and clearly label them as maximum or minimum.

2. (20 points) Consider the region bounded by $x = y^2$ and $x = 1$ with a density of $\delta(x, y) = y^3 + x + 2$.
   (a) Sketch the region in question.
   (b) Your region should be symmetrical with respect to the x-axis. However, the center of mass is not located on the x-axis, but rather at the point $(\frac{57}{91}, \frac{3}{91})$. Explain how this is possible.
   (c) Set up BUT DO NOT EVALUATE the double integral to find the moment of inertia about the y-axis.
   (d) Set up BUT DO NOT EVALUATE the equivalent integral as the one in (c) with the order of integration reversed.

3. (15 points) Suppose the elevation on a hill is given by $f(x, y) = 100 - y^2 - 4x^2$.
   (a) Sketch what a road would look like (when viewed from above) if it was to be built on the hill at a constant elevation of 92 feet.
   (b) In which (horizontal) direction will the rain run off at the point $(1, 2)$?
   (c) Sketch the direction you found in (b), on your sketch in (a), and explain how it relates to the shape of the road. (Note that $(1, 2)$ is at an elevation of 92 feet and is, therefore, on the road.)

4. (15 points) Convert the following integral to cylindrical coordinates BUT DO NOT EVALUATE it:

$$\int_{-2}^{2} \int_{-\sqrt{4-z^2}}^{\sqrt{4-z^2}} \int_{y^2+z^2}^{4} (y^2 + z^2)^{\frac{3}{2}} \, dx \, dy \, dz$$

   [Hint: do not use the standard cylindrical coordinates; rather, treat x as the vertical coordinate and y, z as the horizontal coordinates (the conversions are just the same, but renamed).]

5. (20 points) Use a coordinate system other than rectangular to set up BUT NOT EVALUATE the integral(s) for finding the volume of the solid given by the region below $x^2+y^2+z^2=4z$ and above $z=\sqrt{x^2+y^2}$.

6. (10 points) Find the area of the ellipse $x^2+\frac{y^2}{4}=1$ by using the transformation $x=r\cos(\theta)$, $y=2r\sin(\theta)$.

7. (2 points, EXTRA CREDIT) In hono(u)r of the recent holiday, which of the following phrases comes from the Declaration of Independence? (For extra extra credit, source all quotes — no partial credit!)

   (a) The land of the free and the home of the brave
   (b) With liberty and justice for all
   (c) We the people of the United States
   (d) We hold these truths to be self-evident
   (e) Government of the people, by the people, for the people
   (f) You have to let it all go . . . fear, doubt, and disbelief. Free your mind.

— Useful and interesting formulae —

$$\text{proj}_{\mathbf{A}}\mathbf{B}=\left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d=\frac{|\vec{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d=\left|\vec{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

$$\mathbf{T}=\frac{d\mathbf{r}}{ds}=\frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N}=\frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|}=\frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B}=\mathbf{T}\times\mathbf{N}$$

$$\kappa=\left|\frac{d\mathbf{T}}{ds}\right|=\frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} \qquad \tau=-\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a}=a_T\mathbf{T}+a_N\mathbf{N} \qquad \text{where} \qquad a_T=\frac{d}{dt}|\mathbf{v}|, \qquad a_N=\kappa|\mathbf{v}|^2=\sqrt{|\mathbf{a}|^2-a_T^2}$$

$$|E(x,y)|\le\frac{M}{2}\left(|x-x_0|+|y-y_0|\right)^2 \qquad \text{where} \qquad |f_{xx}|,|f_{xy}|,|f_{yy}|\le M$$

$$\begin{aligned} f(x,y) &= f(0,0)+\left(\left.\frac{\partial f}{\partial x}\right|_{(0,0)}x+\left.\frac{\partial f}{\partial y}\right|_{(0,0)}y\right) \\ &\quad +\frac{1}{2}\left(\left.\frac{\partial^2 f}{\partial x^2}\right|_{(0,0)}x^2+\left.\frac{\partial f^2}{\partial x\partial y}\right|_{(0,0)}xy+\left.\frac{\partial f}{\partial y^2}\right|_{(0,0)}y^2\right) \\ &\quad +\ldots+\frac{1}{n!}\left.\left(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)^n\right|_{(0,0)}f \end{aligned}$$

$$\nabla f=\lambda\nabla g \qquad g=0$$

$$M = \iint_R \delta(x,y)\,dA \qquad M_x = \iint_R y\delta(x,y)\,dA \qquad M_y = \iint_R x\delta(x,y)\,dA$$

$$(\bar{x},\bar{y}) = \left(\frac{M_y}{M},\frac{M_x}{M}\right)$$

$$I_x = \iint_R y^2\delta(x,y)\,dA \qquad I_y = \iint_R x^2\delta(x,y)\,dA \qquad I_0 = \iint_R (x^2+y^2)\delta(x,y)\,dA$$

$$R_x = \sqrt{I_x/M} \qquad R_y = \sqrt{I_y/M}$$

$$\begin{array}{rcl} x &=& r\cos(\theta) \\ y &=& r\sin(\theta) \\ z &=& z \end{array} \qquad \begin{array}{rcl} r &=& \rho\sin(\phi) \\ z &=& \rho\cos(\phi) \\ \theta &=& \theta \end{array} \qquad \begin{array}{rcl} x &=& \rho\sin(\phi)\cos(\theta) \\ y &=& \rho\sin(\phi)\sin(\theta) \\ z &=& \rho\cos(\phi) \end{array}$$

$$x^2+y^2+z^2=\rho^2 \qquad x^2+y^2=r^2=\rho^2\sin^2(\phi)$$

$$dV = dx\,dy\,dz = r\,dr\,d\theta\,dz = \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$$

$$\iint_R f(x,y)\,dx\,dy = \iint_{R'} f(x(u,v),y(u,v))|J(u,v)|\,du\,dv \qquad J(u,v) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

$$\begin{array}{rcl} \iiint_V f(x,y)\,dx\,dy\,dz &=& \iiint_{V'} f(x(u,v,w),y(u,v,w),z(u,v,w))|J(u,v,w)|\,du\,dv\,dw \\ J(u,v,w) &=& \left|\begin{array}{ccc} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{array}\right| \end{array}$$