
INSTRUCTIONS: Computers, calculators, books, notes, flying monkeys, *etc.* are not permitted. Some (possibly) useful formulae are attached. Write your name, your instructor's name, and the color of your exam sheet on the front of your bluebook. Work all problems. Start each problem on a **new page**. **Show your work clearly, label your axes, and** box **your final answer**. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

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1. (20 points) Some Calc III questions that do not relate to each other.
   (a) Find a unit vector orthogonal to $\mathbf{A} = 2\hat{\imath} + \hat{k}$, $\mathbf{B} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$.
   (b) Find the parametric equation of a line through $(2, -1, -3)$ and $(0, 2, -3)$.
   (c) Find an equation for the plane containing the point $(0, -5, 1)$ with a normal vector $4\hat{\imath} + \hat{\jmath} - 2\hat{k}$.
   (d) Sketch the following surfaces. Label the axes.
      i. $y^2 + z^2 = 1$ ii. $y = 2$ iii. $f(x, y) = \sqrt{x^2 + y^2}$

2. (10 points) Find the flux of $\mathbf{F}(x, y, z) = (z - y^3)\hat{\imath} + (2y - \sin(z))\hat{\jmath} + (x^2 - z)\hat{k}$ out of the region bounded by $x^2 + y^2 = 3$, $z = 0$, and $z = 1$.

3. (15 points) Compute the work done by the force field $\mathbf{F}(x, y) = y\hat{\imath} - x\hat{\jmath}$ acting on an object as it moves along the path $y = x^2 - 1$ from $(0, -1)$ to $(2, 3)$.

4. (10 points) True or False. Write out the FULL WORD "True" if the statement is true or "False" if the statement is false. No justification needed.
   (a) If $\mathbf{F}$ is conservative, then $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$.
   (b) If $\mathbf{F}$ is conservative, then $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path.
   (c) $\mathbf{F} = \hat{\imath} + \hat{\jmath} + \hat{k}$ is conservative.
   (d) $f(x, y, z) = x + y + z - \pi$ is a potential function for $\mathbf{F} = \hat{\imath} + \hat{\jmath} + \hat{k}$.
   (e) If $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of path, then $\mathbf{F}$ is conservative.

5. (10 points) Evaluate $\displaystyle\oint_C [7y - e^{\sin(x)}]\, dx + [15x - \sin(y^3 + 8y)]\, dy$ where C is the circle of radius 3 centered at the point $(5, -7)$.

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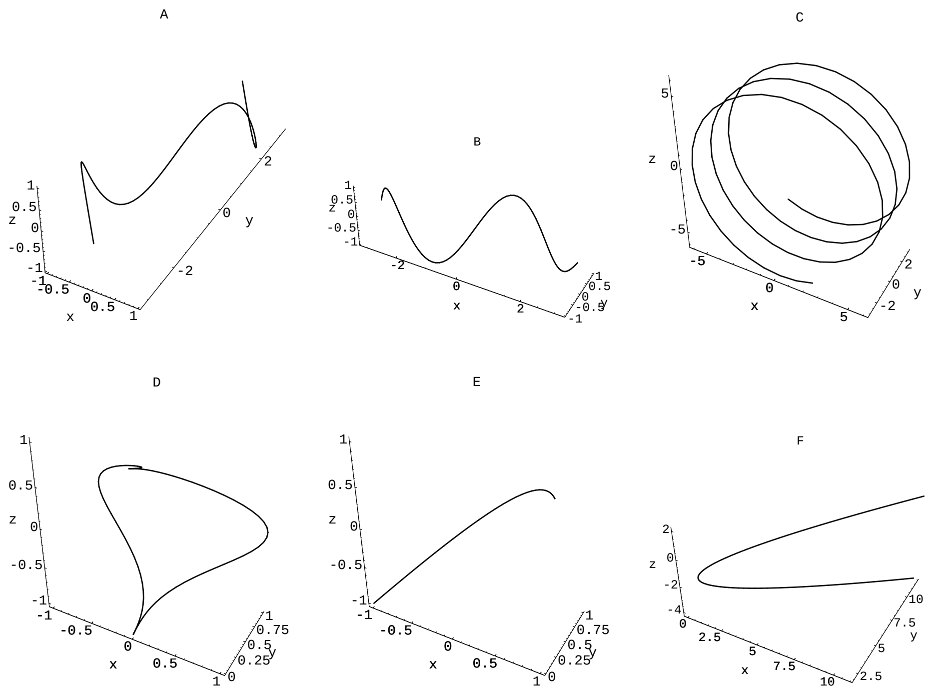
6. (20 points) More Calc III questions that do not relate to each other.

   (a) Match the vector-valued function with its graph. In each graph, $t \in [-\pi, \pi]$.

   i. $\mathbf{r}(t) = 6\sin(\pi t)\hat{\imath} + t\hat{\jmath} + 6\cos(\pi t)\hat{k}$

   ii. $\mathbf{r}(t) = \sin^5(t)\hat{\imath} + \sin^2(t)\hat{\jmath} + \cos(t)\hat{k}$

   iii. $\mathbf{r}(t) = (t^2+1)\hat{\imath} + (t^2+2)\hat{\jmath} + (t-1)\hat{k}$



   (b) Find the limit of $\displaystyle\lim_{(x,y)\to(0,2)} \left(\frac{3x}{y^2+1}\right)$ or explain why it does not exist.

   (c) Let $f(x,y) = 2x^4y + 3x^2y^2$. Find f_{xx} , f_{yy} and f_{xy}.

   (d) Let $f(x,y) = 4x^2 - y$ with $x(u,v) = u^3v + \sin(u)$, and $y(u,v) = 4v^2$. Find $\dfrac{\partial f}{\partial v}$

7. (15 points) Compute the circulation of $\mathbf{F}(x,y,z) = e^{z^2}\hat{\imath} + 4xz\hat{\jmath} + 8y\sin(x)\hat{k}$ around the curve C given by the intersection of the hemisphere $z = \sqrt{5 - x^2 - y^2}$ and the plane $z = 1$.

— Useful and interesting formulae —

$$\text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\vec{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\vec{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

$$\mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N} \qquad \text{where} \qquad a_T = \frac{d}{dt}|\mathbf{v}|, \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$

$$|E(x,y)| \le \frac{M}{2}\left(|x-x_0| + |y-y_0|\right)^2 \qquad \text{where} \qquad |f_{xx}|, |f_{xy}|, |f_{yy}| \le M$$

$$\begin{aligned} f(x,y) &= f(0,0) + \left(\left.\frac{\partial f}{\partial x}\right|_{(0,0)} x + \left.\frac{\partial f}{\partial y}\right|_{(0,0)} y\right) \\ &\quad + \frac{1}{2}\left(\left.\frac{\partial^2 f}{\partial x^2}\right|_{(0,0)} x^2 + \left.\frac{\partial f^2}{\partial x\partial y}\right|_{(0,0)} xy + \left.\frac{\partial f}{\partial y^2}\right|_{(0,0)} y^2\right) \\ &\quad + \ldots + \frac{1}{n!}\left.\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^n\right|_{(0,0)} f \end{aligned}$$

$$\nabla f = \lambda\nabla g \qquad g = 0$$

$$M = \iint_R \delta(x,y)\,dA \qquad M_x = \iint_R y\delta(x,y)\,dA \qquad M_y = \iint_R x\delta(x,y)\,dA$$

$$(\bar{x},\bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right)$$

$$I_x = \iint_R y^2\delta(x,y)\,dA \qquad I_y = \iint_R x^2\delta(x,y)\,dA \qquad I_0 = \iint_R (x^2+y^2)\delta(x,y)\,dA$$

$$R_x = \sqrt{I_x/M} \qquad R_y = \sqrt{I_y/M}$$

$$\begin{aligned} x &= r\cos(\theta) & r &= \rho\sin(\phi) & x &= \rho\sin(\phi)\cos(\theta) \\ y &= r\sin(\theta) & z &= \rho\cos(\phi) & y &= \rho\sin(\phi)\sin(\theta) \\ z &= z & \theta &= \theta & z &= \rho\cos(\theta) \end{aligned}$$

$$x^2 + y^2 + z^2 = \rho^2 \qquad x^2 + y^2 = r^2 = \rho^2\sin^2(\phi)$$

$$dV = dx\,dy\,dz = r\,dr\,d\theta\,dz = \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$$

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$$\iint_R f(x,y)\,dx\,dy = \iint_{R'} f(x(u,v),y(u,v))|J(u,v)|\,du\,dv \qquad J(u,v) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

$$\begin{aligned} \iiint_V f(x,y)\,dx\,dy\,dz &= \iiint_{V'} f(x(u,v,w),y(u,v,w),z(u,v,w))|J(u,v,w)|\,du\,dv\,dw \\ J(u,v,w) &= \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} \end{aligned}$$

Vector identities: $\nabla \times (\nabla f) = \mathbf{0}$, $\nabla \cdot (\nabla \times \mathbf{F}) = 0$

$$\begin{aligned} \text{Work: } \int_C \mathbf{F}\cdot\mathbf{T}\,ds &= \int_C \mathbf{F}\cdot d\mathbf{r} = \int_C M\,dx + N\,dy \\ \text{Flux: } \int_C \mathbf{F}\cdot\mathbf{n}\,ds &= \int_C M\,dy - N\,dx \end{aligned}$$

Surface area of surface given by $f(x,y,z) = c$: $\displaystyle\iint_S d\sigma = \iint_R \frac{|\nabla f|}{|\nabla f \cdot \mathbf{p}|}\,dA$

Flux of $\mathbf{F}$ through surface given by $f(x,y,z) = c$: $\displaystyle\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iint_R \frac{\pm\mathbf{F}\cdot\nabla f}{|\nabla f \cdot \mathbf{p}|}\,dA$

Green's Theorem:

$$\begin{aligned} \text{Circulation} &= \oint_C \mathbf{F}\cdot\mathbf{T}\,ds = \iint_R (\nabla\times\mathbf{F})\cdot\hat{k}\,dA = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA \\ \text{Flux} &= \oint_C \mathbf{F}\cdot\mathbf{n}\,ds = \iint_R \nabla\cdot\mathbf{F}\,dA = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dA \end{aligned}$$

Stokes's Theorem:

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C M dx + N dy + P dz = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$$

Divergence Theorem of Gauss:

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,d\sigma = \iiint_D \nabla\cdot\mathbf{F}\,dV$$