
INSTRUCTIONS: Computers, calculators, books, notes, flying monkeys, *etc.* are not permitted. Some (possibly) useful formulae are attached. Write your name, your instructor's name, and the color of your exam sheet on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show **all** your work clearly and box your final answer. A correct answer with incorrect or no supporting work may **receive no credit**, while an incorrect answer with relevant work may receive partial credit.

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1. (20 points) Consider the function $f(x, y) = \dfrac{x^2\sqrt{1 - xy}}{x^2 + y^2}$.

   (a) What is the domain of $f(x, y)$? Sketch it; be sure to show which boundary points are included in the domain and which are not.

   (b) Describe the domain of f in terms of open/closedness and boundedness.

   (c) Find the limit of $f(x, y)$ as $(x, y) \to (0, 0)$ along the y-axis.

   (d) Find the limit of $f(x, y)$ as $(x, y) \to (0, 0)$ along the line $y = x$.

   (e) What, if anything, do your answers to (c) and (d) tell you about $\lim\limits_{(x,y)\to(0,0)} f(x, y)$?

   (f) Is $f(x, y)$ continuous at $(0, 0)$?

2. (20 points) A space probe moves along the path given by $\mathbf{r}(t) = [\cos(t) + t\sin(t)]\hat{\imath} + 3\hat{\jmath} + [\sin(t) - t\cos(t)]\hat{k}$.

   (a) Find the points in space, P_0 and P_1, where the probe is located at times $t = 0$ and $t = \pi$.

   (b) How many times further does the probe travel in going from P_0 to P_1 than if it travelled along a direct path?

   (c) The TNB frame for the probe's path is given by: $\mathbf{T} = \cos(t)\hat{\imath} + \sin(t)\hat{k}, \mathbf{N} = -\sin(t)\hat{\imath} + \cos(t)\hat{k}, \mathbf{B} = \hat{\jmath}$. Express the vector $\mathbf{r}(t)$ in the form $\mathbf{r}(t) = c_T\mathbf{T} + c_N\mathbf{N} + c_B\mathbf{B}$ (where c_T, c_N, c_B are scalars).

3. (20 points) Find the equation of the plane containing the three points $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$ (where a, b, c are non-zero).

4. (20 points) Consider a curve in space described by $\mathbf{r}(t)$. Although you do not know the form of the function $\mathbf{r}$, you do know that, at a given time t^*, $\mathbf{r}(t^*) = \hat{\imath} - 2\hat{\jmath} - \hat{k}$, $\mathbf{v}(t^*) = 2\hat{\jmath} - \hat{k}$ and $\mathbf{a}(t^*) = 2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$. You also know that the speed along the curve is constant. Calculate the following quantities (at $t = t^*$) or explain why it is not possible to calculate them.

   (a) The unit tangent $\mathbf{T}$.

   (b) The unit normal $\mathbf{N}$.

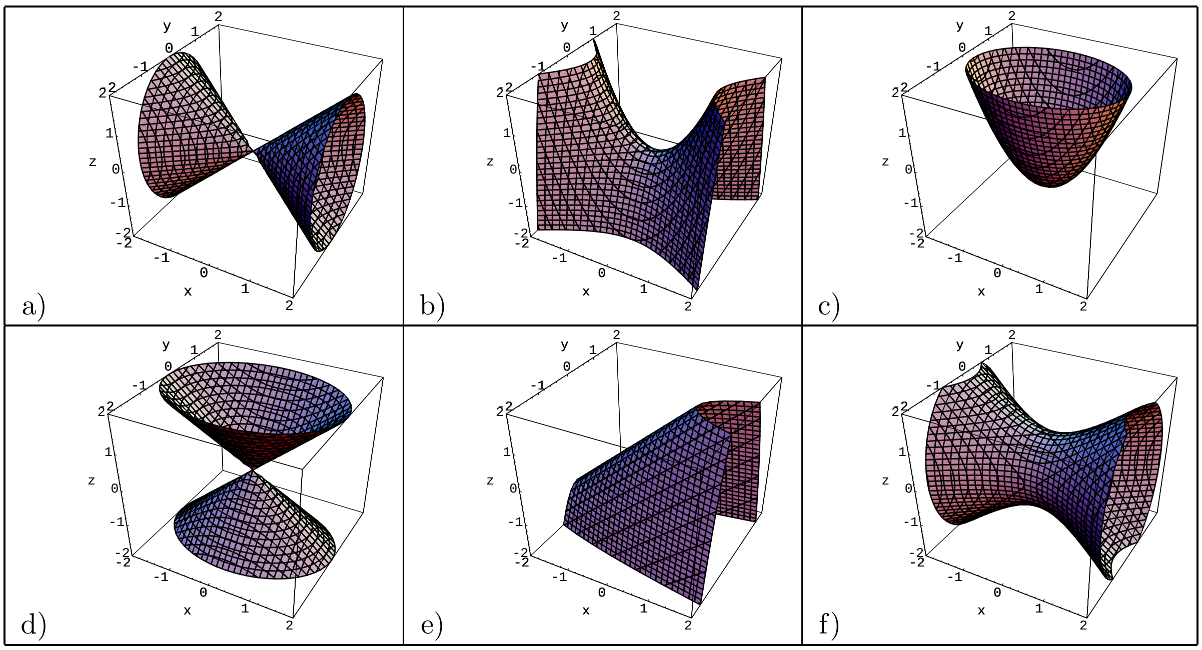
   (c) The unit binormal $\mathbf{B}$.

   (d) The curvature κ.

   (e) The torsion τ.

   (f) Write the acceleration as $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$.

5. (20 points) Match each of the pictures shown (a)-(f) with one of the equations below. (Note: there are more equations than pictures, so three equations will be unused.) No work need be shown for this problem.



(1)	$2x^2 - y^2 - z^2 = 0$	(2)	$x - 2y^2 - z = -1$	(3)	$x^2 - 2y^2 - z^2 = 0$
(4)	$x^2 + 2y^2 - z^2 = 0$	(5)	$x^2 - 2y^2 - z = 0$	(6)	$x^2 - 2y^2 - z^2 = -1$
(7)	$x^2 - 2y^2 + z^2 = 0$	(8)	$x^2 + 2y^2 - z = 1$	(9)	$x^2 - 2y^2 - z^2 = 1$

— Useful and interesting formulae —

$$\text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\vec{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\vec{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

$$s(t) = \int_{t_0}^{t} |\mathbf{v}(u)|\,du \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{[\dot{x}^2+\dot{y}^2]^{3/2}} \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N} \qquad \text{where} \qquad a_T = \frac{d}{dt}|\mathbf{v}|, \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$