
INSTRUCTIONS: Computers, calculators, books, notes, flying monkeys, *etc.* are not permitted. Some (possibly) useful formulae are attached. Write your name and your instructor's name on the front of your bluebook. Work all problems. Start each problem on a **new page**. Unless otherwise stated, show **all** your work clearly and box your final answer; a correct answer with incorrect or no supporting work will **receive no credit**, while an incorrect answer with relevant work may receive partial credit.

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1. (20 points) Multi-choice; choose the best answer. You do not need to show your work for this question.

(a) Consider the expression $\mathbf{A} \cdot (\quad) = \mathbf{A} \cdot \mathbf{B}$. What can go in $(\quad)$?

i. only $\mathbf{B}$
ii. any non–zero vector
iii. only vectors parallel to $\mathbf{B}$
iv. only vectors with magnitude $|\mathbf{B}|$
v. an infinite number of vectors (although not just any vector will work.)

(b) Consider the expressions

$$1 : (\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) \qquad 2 : (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$
$$3 : (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \times (\mathbf{B} \cdot \mathbf{C}) \qquad 4 : (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = (\mathbf{A} \cdot \mathbf{C}) \times (\mathbf{B} \cdot \mathbf{C})$$
$$5 : (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{A} = 0 \qquad 6 : (\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{A} \times \mathbf{C}) = (\mathbf{B} \cdot \mathbf{C})\ |\mathbf{A}|^2$$

Which of the expressions are true?

i. 1, 2, 3, 6
ii. 4, 6
iii. 2, 5
iv. 1, 3, 4, 5
v. 2, 3, 4

(c) If $\mathbf{r}(t) = f(t)\hat{\jmath} + g(t)\hat{k}$ then

i. $\mathbf{a}(t)$ is in the $\hat{\jmath}$-$\hat{k}$ plane; $\mathbf{B}$ is in the $\hat{\imath}$ direction
ii. $\mathbf{a}(t)$ is in the $\hat{\jmath}$-$\hat{k}$ plane; $\mathbf{B}$ is in the $\hat{\jmath}$-$\hat{k}$ plane
iii. $\mathbf{a}(t)$ is in the $\hat{\imath}$ direction; $\mathbf{B}$ is in the $\hat{\imath}$ direction
iv. $\mathbf{a}(t)$ is in the $\hat{\imath}$ direction; $\mathbf{B}$ is in the $\hat{\jmath}$-$\hat{k}$ plane
v. None of the above.

(d) If a satellite goes around the Earth in a circular path, then

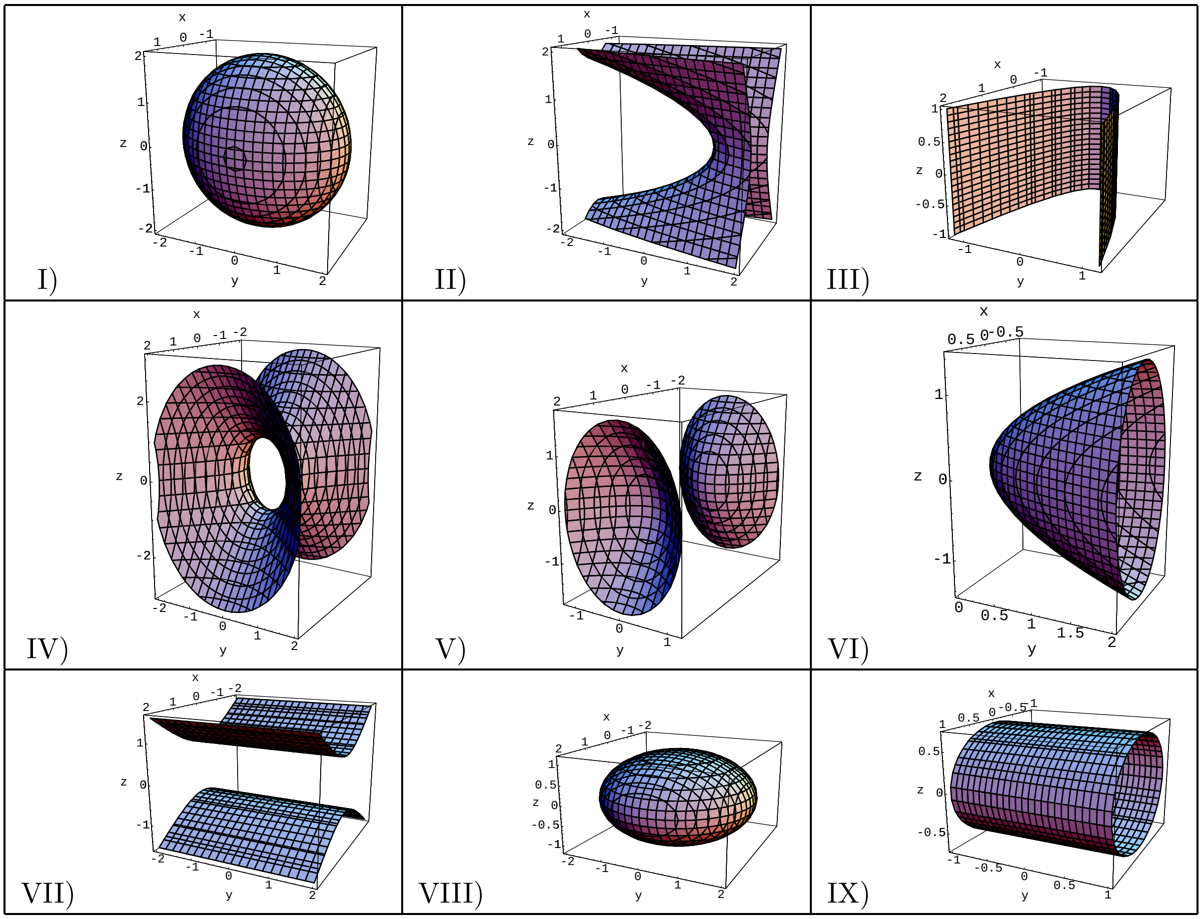
i. $\mathbf{T}$ always points towards the Earth
ii. $\mathbf{N}$ always points towards the Earth
iii. $\mathbf{B}$ always points towards the Earth
iv. $\kappa = 0$
v. the satellite's acceleration is purely tangential

2. (20 points) Bela goes down a slide at the water park. She starts at the top of the slide at time $t = 0$; the bottom of the slide is at $z = 0$. Her path is described by the vector-valued function $\mathbf{r}(t) = \frac{2}{3}t^3\hat{\imath} + t\hat{\jmath} + (4 - t^2)\hat{k}$.
   (a) What is the location of the top of the slide? What is the location of the bottom of the slide, and how long does it take Bela to get there?
   (b) Calculate her speed as a function of t. Give your answer in the simplest possible form.
   (c) What is her average speed (from beginning to end)?
   (d) What is her direction of motion at the top of the slide? At the bottom of the slide?

3. (20 points) Three microphones are lowered into the water to record dolphins. The mic's are located at $A(2, 1, -3)$, $B(1, 1, -1)$, and $C(5, 2, -4)$.
   (a) Find the equation of the plane containing A, B, and C.
   (b) What is the area of the triangle between the three mic's?
   (c) Mic A detects a dolphin in the direction of $\hat{\imath} - 4\hat{\jmath}$ (from it); mic B detects the same dolphin in the direction of $\hat{\imath} - 2\hat{\jmath} - \hat{k}$. Where is the dolphin?
   (d) In what direction should C detect the dolphin?
   (e) From the dolphin's perspective, what is the angle between A and B? (You can leave your answer in the form of a trigonometric function.)
   (f) The dolphin is moving in a straight-line path in the direction of $\mathbf{v} = -3\hat{\imath} + 3\hat{\jmath} - \hat{k}$. What is the dolphin's location as it crosses the plane containing all three mic's?

4. (20 points) Let $f(x, y) = \sqrt{x^2 - y^2}$.
   (a) What is the domain of $f(x, y)$? Sketch it.
   (b) Describe the domain in terms of open/closedness and boundedness.
   (c) Is $f(x, y)$ continuous at $(x, y) = (0, 0)$?
   (d) The vector $\mathbf{n} = -5\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ is normal to the surface $z = f(x, y)$ at $(x, y) = (5, 3)$. Find the equation of the plane that approximates f near $(5, 3)$ — *i.e.* the plane that has the same normal and goes through same point.

5. (20 points) Match each of the four equations with one of the graphs shown. In each graph, z is vertical and x is coming out of the page; the scale is marked on the axes. No work need be shown for this problem.



(a)	$2x^2 + y^2 + z^2 = 4$	(b)	$x^2 - 2z^2 = -1$
(c)	$x^2 - 2y^2 - z^2 = 1$	(d)	$3x^2 - y + z^2 = 0$

— Useful and interesting formulae —

$$\text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\vec{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\vec{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

$$s(t) = \int_{t_0}^{t} |\mathbf{v}(u)|\,du \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{[\dot{x}^2+\dot{y}^2]^{3/2}} \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N} \qquad \text{where} \qquad a_T = \frac{d}{dt}|\mathbf{v}|, \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$