#1 a) $\vec{F}(x,y,z) = \underbrace{(x+2y+az)}_{M}\hat{i} + \underbrace{(bx-3y-z)}_{N}\hat{j} + \underbrace{(4x+cy+2z)}_{P}\hat{k}$

A) $\frac{\partial P}{\partial y} = c \qquad \frac{\partial N}{\partial z} = -1 \quad \Rightarrow \quad c=-1 \qquad$ since $\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}$ for a conservative field

B) $\frac{\partial P}{\partial x} = 4 \qquad \frac{\partial M}{\partial z} = a \quad \Rightarrow \quad a=4 \qquad$ since $\frac{\partial P}{\partial x} = \frac{\partial M}{\partial z}$ " " " "

c) $\frac{\partial N}{\partial x} = b \qquad \frac{\partial M}{\partial y} = 2 \quad \Rightarrow \quad b=2 \qquad$ since $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$ " " " "

Hence: $\boxed{a=4 \qquad b=2 \qquad c=-1}$

b) $\vec{F}(x,y,z) = \underbrace{xy}_{M}\hat{i} \underbrace{-z}_{N}\hat{j} + \underbrace{x^2}_{P}\hat{k}$

If $\vec{F}$ is conservative, then $\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}$, $\frac{\partial P}{\partial x} = \frac{\partial M}{\partial z}$, $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$.

So:

$\frac{\partial P}{\partial y} = 0$ and $\frac{\partial N}{\partial z} = -1$

Hence $\frac{\partial P}{\partial y} \neq \frac{\partial N}{\partial z} \quad \Rightarrow$

> $\vec{F}$ is not a conservative field.
> And since $\vec{F}$ is not conservative, there is no f such that $\vec{F} = \nabla f$.

c) $\int_C \vec{F}\cdot d\vec{r} = \int_0^1 M\,dx + N\,dy + P\,dz$

$x = t^2 \Rightarrow dx = 2t\,dt$

$y = 2t \Rightarrow dy = 2\,dt$

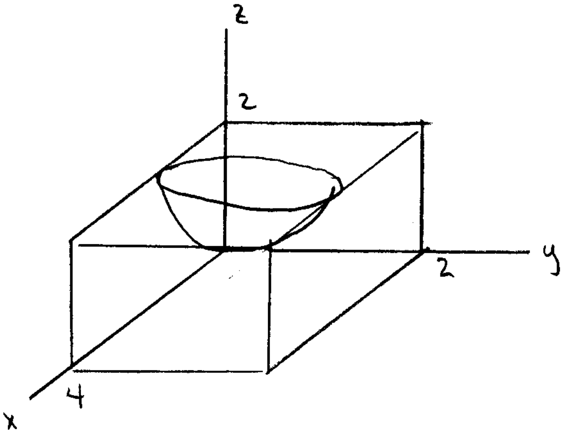
$z = t^3 \Rightarrow dz = 3t^2\,dt$

$$= \int_0^1 (2t^3)\,2t\,dt - t^3\,2\,dt + t^4\,3t^2\,dt$$

$$= \int_0^1 (4t^4 - 2t^3 + 3t^6)\,dt$$

$$\boxed{= \frac{4}{5} - \frac{1}{2} + \frac{3}{7}}$$

\# 2a)



b) Number of bits = $\iiint_D \delta(x,y,z)\,dV$ D = original box

$$= \int_0^4 \int_0^2 \int_0^2 (2+z^2)\,dz\,dy\,dx$$

$$= 8\int_0^2 (2+z^2)\,dz$$

$$\boxed{= 32 + \frac{64}{3} = \frac{160}{3}}$$

c) $dV = dx\,dy\,dz = |J|\,du\,dv\,dw$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$x = u + x_0$
$y = v + y_0$
$z = w + z_0$

$$\boxed{\text{So: } dV = dx\,dy\,dz = |J|\,du\,dv\,dw = du\,dv\,dw}$$

d)

In spherical coordinates:

$$\int_0^{2\pi}\int_{\pi/2}^{\pi}\int_0^{1}\left[2+(2+\rho\cos\phi)^2\right]\rho^2\sin\phi \, d\rho \, d\phi \, d\theta$$

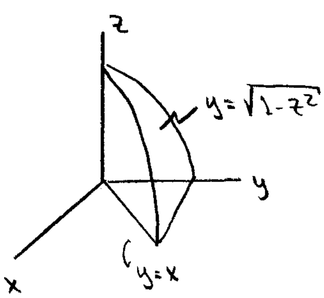
In cylindrical coordinate:

$$\int_0^{2\pi}\int_{-1}^{0}\int_0^{\sqrt{1-z^2}}\left[2+(2+z)^2\right] r \, dr \, dz \, d\theta$$

or

$$\int_0^{2\pi}\int_1^{2}\int_0^{\sqrt{1-z^2}}(2+z^2)\, r \, dr \, dz \, d\theta$$

#3)



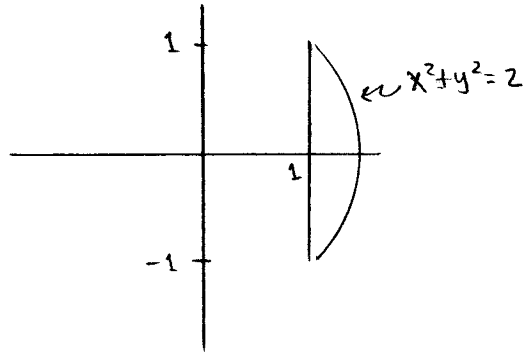
$$r\sin\theta=\sqrt{1-z^2} \Rightarrow r=\csc\theta\sqrt{1-z^2}$$

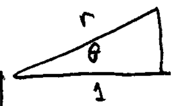
$$I=\int_0^{1}\int_{\pi/4}^{\pi/2}\int_0^{\sqrt{1-z^2}\csc\theta} 2zr \, dr \, d\theta \, dz$$

$$=\frac{1}{4}$$

4)

$$I = \int_{-1}^{1} \int_{1}^{\sqrt{2-y^2}} \frac{x}{x^2+y^2}\, dx\, dy$$





$r = \sec\theta$

$$\boxed{\Rightarrow \quad I = \int_{-\pi/4}^{\pi/4} \int_{\sec\theta}^{\sqrt{2}} \cos\theta \, dr\, d\theta}$$

#5) $x = au\cos v$

$y = bu\sin v$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \Rightarrow u^2\cos^2 v + u^2 \sin^2 v \leq 1 \Rightarrow u^2 \leq 1$$

$u \geq 0$ was given, so new region is: $\begin{matrix} 0 \leq u \leq 1 \\ 0 \leq v \leq 2\pi \end{matrix}$

$$J(x,y) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} a\cos v & -au\sin v \\ b\sin v & bu\cos v \end{vmatrix} = abu$$

Thus:
$$\boxed{\iint_R f(x,y)\, dA = ab \int_0^{2\pi} \int_0^1 f(au\cos v, au\sin v)\, u\, du\, dv}$$

b) $$\text{Area} = ab \int_0^{2\pi} \int_0^1 u \, du \, dv$$

$$\boxed{= \pi ab}$$

c) If $a = b$ then the region is a circle and has area $\pi R^2 = \pi a^2 = \pi b^2$ as it should