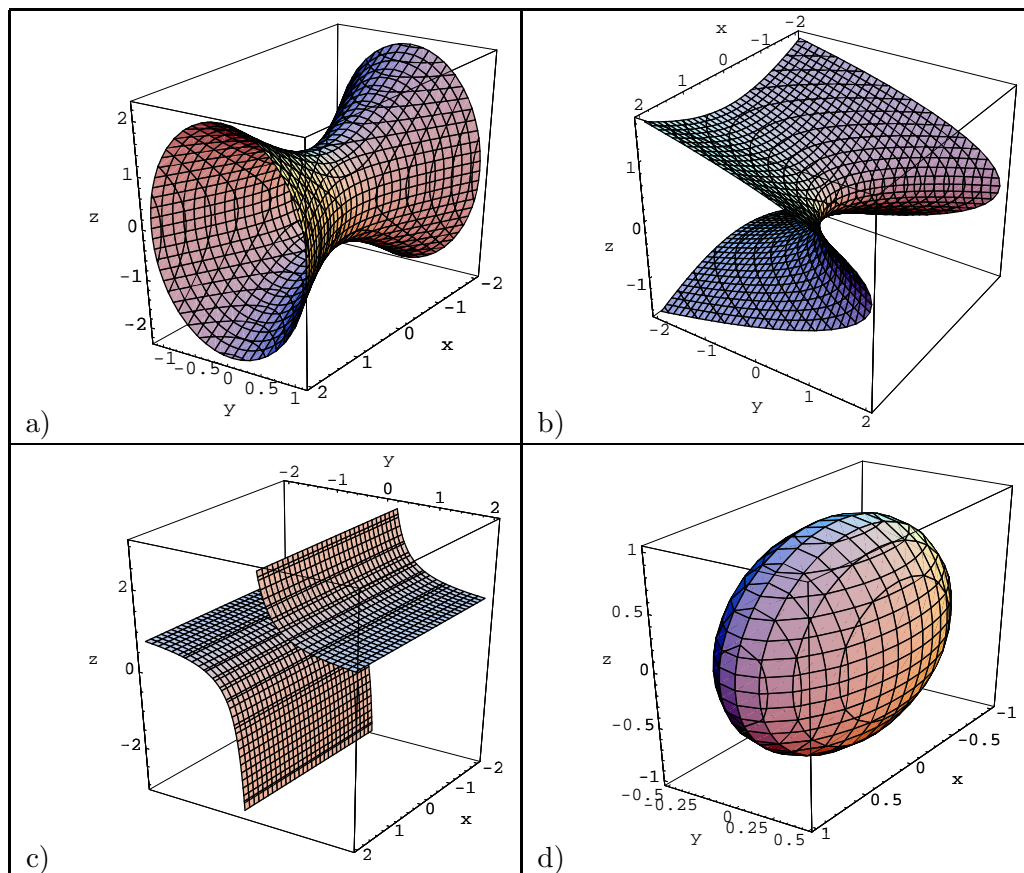

INSTRUCTIONS: Computers, calculators, books, notes, flying monkeys, *etc.* are not permitted. Some (possibly) useful formulae are attached. Write your name, your instructor's name, and the color of your exam sheet on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1.
 - (a) Calculate the equation for the plane through the points $A(-1, 1, 1)$, $B(1, -1, 1)$, $C(1, 1, -1)$.
 - (b) Calculate the distance between the point $P(-1, -1, 0)$ and the plane from (a).
 - (c) Sketch and describe (in words) the curve of intersection between the plane $x + y = 0$ and the surface $x^2 + y^2 - z = 1$.
 - (d) Find a parametrization of the curve in (c).
2. Prove or disprove the following statements (be sure to state clearly whether you believe them to be TRUE or FALSE):
 - (a) If $\mathbf{A} \cdot \mathbf{B} = \mathbf{C} \cdot \mathbf{B}$ and $\mathbf{B} \neq \mathbf{0}$, then $\mathbf{A} = \mathbf{C}$.
 - (b) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{A} = 0$.
 - (c) If an object moves with constant speed, its acceleration has no tangential component and a constant normal component.
3. A particle moves along the ellipse, $\mathbf{r}(t) = \cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}$.
 - (a) Find the minimum and maximum magnitudes of its velocity and acceleration, *i.e.* the minimum and maximum values of $|\mathbf{v}|$ and $|\mathbf{a}|$.
 - (b) Find the curvature as a function of t .
 - (c) Find the minimum and maximum values of the curvature. What is the position of the particle at these values?
4. A curve is given by $\mathbf{r}(t) = \cos(t)\mathbf{i} + 2t\mathbf{j} + \sin(t)\mathbf{k}$.
 - (a) Calculate the tangential, normal and binormal directions of this curve, *i.e.* calculate \mathbf{T} , \mathbf{N} and \mathbf{B} .
 - (b) Calculate the curvature of the curve.
 - (c) Calculate the normal and tangential components of the acceleration.

5. Match each of the pictures shown (a)-(d) with one of the equations below. (Note: there are more equations than pictures, so five equations will be unused.) No work need be shown for this problem.



- (1) $x^2 + 4y^2 - z^2 = 1$ (2) $x^2 - 4y^2 - z^2 = -1$ (3) $2x(y - 1)z = 1$
 (4) $2y = x^2 - 3z^2$ (5) $2y(z - 1) = 1$ (6) $2x = y^2 - 3z^2$
 (7) $x^2 - 4y^2 - z^2 = 1$ (8) $x^2 + 4y^2 + z^2 = 1$ (9) $z = x^2 + 4y^2$

— Useful and interesting formulae —

$$\text{proj}_{\mathbf{A}} \mathbf{B} = \left(\frac{\mathbf{A} \cdot \mathbf{B}}{\mathbf{A} \cdot \mathbf{A}} \right) \mathbf{A} \quad d = \frac{|\vec{PS} \times \mathbf{v}|}{|\mathbf{v}|} \quad d = \left| \vec{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$

$$\hat{\mathbf{T}} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \quad \hat{\mathbf{N}} = \frac{d\hat{\mathbf{T}}/ds}{|d\hat{\mathbf{T}}/ds|} = \frac{d\hat{\mathbf{T}}/dt}{|d\hat{\mathbf{T}}/dt|} \quad \mathbf{B} = \hat{\mathbf{T}} \times \hat{\mathbf{N}}$$

$$\kappa = \left| \frac{d\hat{\mathbf{T}}}{ds} \right| = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} \quad \tau = -\frac{d\mathbf{B}}{ds} \cdot \hat{\mathbf{N}}$$

$$\mathbf{a} = a_T \hat{\mathbf{T}} + a_N \hat{\mathbf{N}} \quad \text{where} \quad a_T = \frac{d}{dt} |\mathbf{v}|, \quad a_N = \kappa |\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$