

**INSTRUCTIONS:** Computers, calculators, books, and crib sheets are not permitted. Write your (1) name, (2) instructor's name, and (3) lecture number (010 or 020) on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

---

1. (20 points) Mark the following statements as **True** or **False**. Note that for a statement to be true, it must be *always* true, whereas for a statement to be false, it only need be false for a single case. Here  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are nonzero vectors (thus  $\mathbf{A} \neq \mathbf{0}$ , etc.),  $c$  is a scalar, and  $\mathbf{q}(t)$  and  $\mathbf{r}(t)$  are differentiable vector-valued functions in three dimensional space. No supporting work need be shown for this problem.
  - (a)  $(\mathbf{A} + \mathbf{B}) \times (\mathbf{A} - \mathbf{B}) = -2(\mathbf{A} \times \mathbf{B})$
  - (b)  $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{A} = \mathbf{B} \cdot (\mathbf{B} \times \mathbf{A})$
  - (c)  $c(\mathbf{A} \times \mathbf{B}) = (c\mathbf{A}) \times (c\mathbf{B})$
  - (d)  $\frac{d}{dt}(\mathbf{q}(t) \cdot \mathbf{r}(t)) = \frac{d\mathbf{q}(t)}{dt} \cdot \mathbf{r}(t) - \frac{d\mathbf{r}(t)}{dt} \cdot \mathbf{q}(t)$
  - (e) If  $\frac{d}{dt}|\mathbf{r}(t)| = 0$  for all  $t$ , then the points on the curve defined by  $\mathbf{r}(t)$  all lie on the surface of a sphere of fixed radius.
  
2. (20 points) Consider the plane  $M$  given by  $x + y + z = 1$  and the points  $A(2, -1, 0)$ ,  $B(-2, 3, 0)$ , and  $C(0, 0, -1)$ .
  - (a) Which of the points  $A$ ,  $B$ , and  $C$  are in the plane  $M$ ?
  - (b) Find the distance from plane  $M$  to each of the points not on  $M$ .
  - (c) Determine the standard equation of the plane  $ABC$  that contains all three points.
  - (d) Determine a parametric equation for the line of intersection between planes  $ABC$  and  $M$ .
  
3. (20 points) Josh, the applied math ski racer, has been blindfolded by his backwards goggles, once again. Being used to skiing this way, he still breaks some ski race records. The path Josh takes down the mountain is given by  $\mathbf{r}(t) = \cosh(t)\mathbf{i} + \left(\frac{3}{5}t\right)\mathbf{j} + \left(20 - \frac{4}{5}t\right)\mathbf{k}$ . Josh starts at the top of the mountain when  $t = 0$  and reaches the bottom of the mountain where the  $\mathbf{k}$  component  $z(t) = 0$ . You should find the following formulae useful.

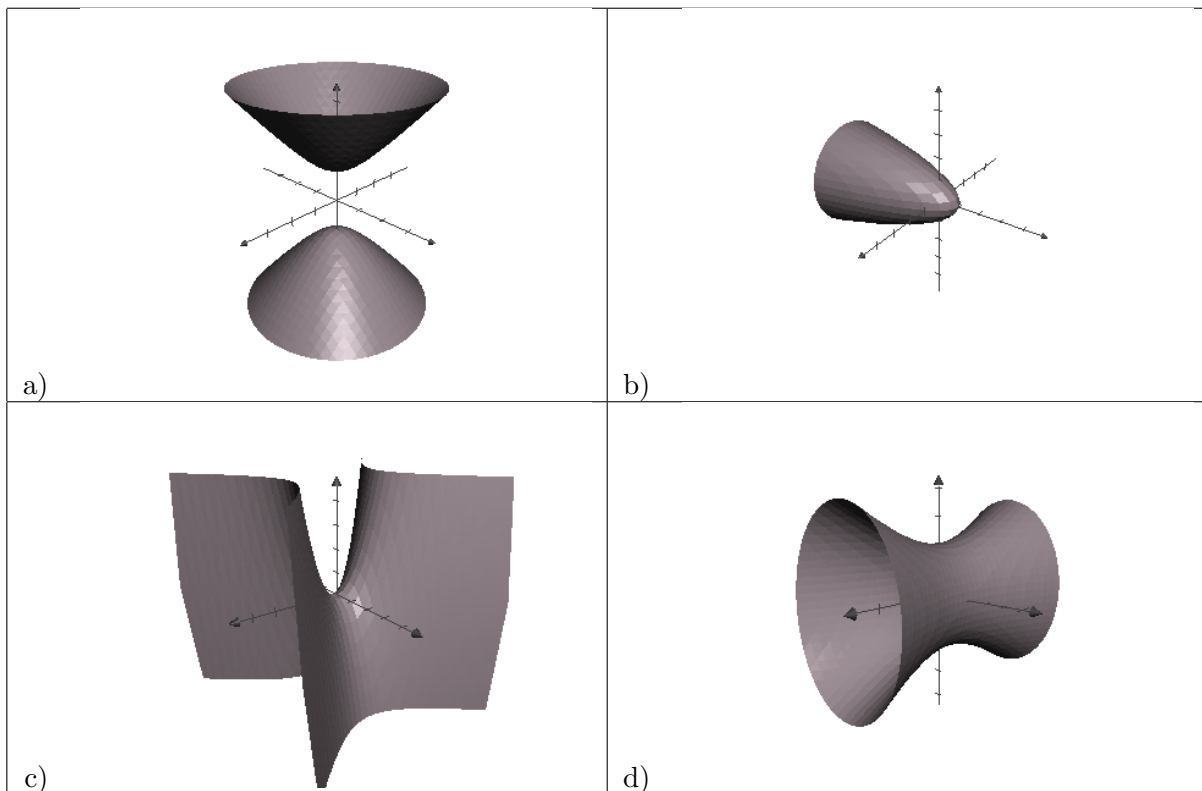
$$\cosh(t) = \frac{e^t + e^{-t}}{2} \qquad \sinh(t) = \frac{e^t - e^{-t}}{2} \qquad \sinh^2(t) + 1 = \cosh^2(t)$$

- (a) How long will it take Josh to reach the bottom of the mountain?
- (b) Give Josh's speed as a function of  $t$ , in its most simplified form. What is Josh's maximum speed as he skis from the top of the mountain to the bottom?
- (c) As he travels down the mountain, is Josh's acceleration ever in the same direction as his velocity? Is his acceleration ever in the same direction as his position vector?
- (d) Calculate the arc length of Josh's path as he travels from the top of the mountain to the bottom.
- (e) What is his average speed as he travels from the top of the mountain to the bottom?

**OVER**

4. (20 points) Consider the position vector of a particle in 3 dimensional space defined by  $\mathbf{r}(t) = t\mathbf{i} - \ln(\cos(t))\mathbf{j} + 10\mathbf{k}$ , for  $0 \leq t \leq \frac{\pi}{2}$ . Perform the following calculations:
- Compute the particle's velocity  $\mathbf{v}(t)$  and the unit tangent vector  $\mathbf{T}$ .
  - Write an integral formula to calculate  $s(t)$ , the distance traveled by the particle from time  $t_0 = 0$  until an arbitrary time  $t$ . **Set up, but do not evaluate this integral.**
  - Compute the principal unit normal vector  $\mathbf{N}$  and unit binormal vector  $\mathbf{B}$ .
  - Determine the curvature  $\kappa$ .
  - Determine the torsion  $\tau$ .
  - At what time(s), if any, is the particle's velocity orthogonal to the acceleration? Parallel?
5. (20 points) Match each of the four surfaces shown below (a–d) with one of the following nine equations (1–9). Note there are more equations than pictures, so some equations will be unused. No work need be shown for this problem.

- |                           |                             |                              |
|---------------------------|-----------------------------|------------------------------|
| (1) $x^2 - y^2 - z^2 = 1$ | (4) $y^2 - z^2 = -1$        | (7) $4x^2 - 4y^2 - z^2 = -4$ |
| (2) $x^2 + y^2 - z = -1$  | (5) $4x^2 + 4y^2 - z^2 = 4$ | (8) $x^2 - y^2 - z = 0$      |
| (3) $x^2 + y + z^2 = 1$   | (6) $x^2 + y^2 - z^2 = -1$  | (9) $x^2 - y^2 + z = 2$      |



### Projections and distances

$$\text{proj}_{\mathbf{A}}\mathbf{B} = \left( \frac{\mathbf{A} \cdot \mathbf{B}}{\mathbf{A} \cdot \mathbf{A}} \right) \mathbf{A} \quad d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|} \quad d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right|$$

### Arc length, frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}| dt \quad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \quad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \quad \mathbf{B} = \mathbf{T} \times \mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N} \quad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N} \quad \kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{|1 + (f'(x))^2|^{3/2}} = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{|\dot{x}^2 + \dot{y}^2|^{3/2}} \quad \tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T} \quad a_T = \frac{d|\mathbf{v}|}{dt} \quad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$