APPM 2350 Exam 1 Fall 2006

1) a) T b) T c) F d) F e) T

2) a) Point A is on plane M because

$$(x+y+z)\Big|_{2,-1,0} = 2-1+0 = 1$$

Point B is on M because

$$(x+y+z)\Big|_{-2,3,0} = -2+3+0 = 1$$

Point C is not on M because

$$(x+y+z)\Big|_{0,0,-1} = 0+0-1 = -1 \neq 1$$

b) Dist from point C to plane M with normal $\underline{n}$ is

$$d = \left| \underline{AC} \cdot \frac{\underline{n}}{|\underline{n}|} \right| = \left| \left((0-2)\hat{i} + (0+1)\hat{j} + (-1-0)\hat{k}\right) \cdot \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}} \right|$$

$$= \left| \frac{-2+1-1}{\sqrt{3}} \right| = 2/\sqrt{3}$$

c) A normal to plane ABC is $\underline{n}_1 = \underline{AC} \times \underline{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -1 \\ -4 & 4 & 0 \end{vmatrix}$

$$= 4\hat{i} + 4\hat{j} - 4\hat{k}$$

Hence $4x + 4y - 4z = D$ where D is found by evaluating at points A, B, or C. Using pt C we have

$$(4x+4y-4z)\Big|_{0,0,-1} = 4 = D$$

Hence plane ABC is $x + y - z = 1$

2) d) Note that points A and B are in both planes M and ABC and thus are on the line of interest.

So $\underline{r}(t) = \underline{OA} + t(\underline{AB})$

$$= (2\hat{i} - \hat{j} + 0\hat{k}) + t((-2-2)\hat{i} + (3+1)\hat{j} + 0\hat{k})$$

$$= (2-4t)\hat{i} + (-1+4t)\hat{j} + 0\hat{k}$$

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3) a) Bottom of mountain is where $\hat{k}$ component of $\underline{r}(t)$ is zero.

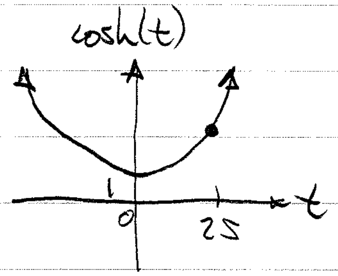
So $20 - \frac{4}{5}t = 0 \Rightarrow t = 20 \cdot \frac{5}{4} = 25$

b) speed $= |\underline{V}| = \left| \sinh t \ \hat{i} + \frac{3}{5}\hat{j} - \frac{4}{5}\hat{k} \right|$

$$= \left(\sinh^2 t + \frac{9}{25} + \frac{16}{25}\right)^{1/2}$$

$$= (\sinh^2 t + 1)^{1/2} = (\cosh^2 t)^{1/2} = |\cosh t|$$

$= \cosh(t)$ since $\cosh(t)$ is never negative.



Max. speed occurs at $t = 25$.

c) $\underline{a} = \cosh(t)\,\hat{i} + 0\,\hat{j} + 0\,\hat{k}$

$\underline{V} = \sinh(t)\,\hat{i} + \frac{3}{5}\,\hat{j} - \frac{4}{5}\,\hat{k}$

$\underline{r} = \cosh(t)\,\hat{i} + \left(\frac{3}{5}t\right)\hat{j} + \left(20 - \frac{4}{5}t\right)\hat{k}$

Is $\underline{a} \parallel \underline{V}$? No, because $\underline{a}$ never has non-zero $\hat{j}$ & $\hat{k}$ components. Or, you can show that $\underline{a} \times \underline{V} \neq \underline{0}$.

Is $\underline{a} \parallel \underline{r}$? No, because the $\hat{j}$ and $\hat{k}$ components of $\underline{r}(t)$ are not both 0 for $0 \le t \le 25$.

Or, you can show that $\underline{a} \times \underline{r} \neq \underline{0}$.

3) d) $s = \int_0^{25} |\underline{v}|\, dt = \int_0^{25} \cosh(t)\, dt = \sinh(t)\Big|_{t=0}^{25} = \sinh(25)$

e) $\bar{s}_{[0,25]} = \dfrac{\text{dist travel.}}{\text{total time}} = \dfrac{\sinh(25)}{25}$

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4) a) $\underline{v} = \hat{i} + \dfrac{\sin(t)}{\cos(t)}\hat{j} + 0\hat{k} \Rightarrow |\underline{v}| = \sqrt{1+\tan^2 t} = |\sec(t)|$

$= \sec(t)$ for $0 \le t \le \frac{\pi}{2}$

so $\hat{T} = \dfrac{\underline{v}}{|\underline{v}|} = \cos(t)\hat{i} + \sin(t)\hat{j} + 0\hat{k}$

b) $s(t) = \int_{\tau=0}^{t} |\underline{v}(\tau)|\, d\tau = \int_{\tau=0}^{\tau=t} \sec(\tau)\, d\tau$

c) $\dfrac{d\hat{T}}{ds} = \kappa \hat{N} = \dfrac{d\hat{T}}{dt}\dfrac{1}{|\underline{v}|} = \underbrace{\left(-\sin(t)\hat{i} + \cos(t)\hat{j} + 0\hat{k}\right)}\dfrac{1}{\sec(t)}$

magnitude of $d\hat{T}/ds$ — unit vector

This vector has a magnitude of 1 so it must be $\hat{N}$ — this must be κ!

$\kappa = \dfrac{1}{\sec(t)} = \cos(t)$

$\hat{N} = -\sin(t)\hat{i} + \cos(t)\hat{j} + 0\hat{k}$

Now $\hat{B} = \hat{T} \times \hat{N} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \cos(t) & \sin(t) & 0 \\ -\sin(t) & \cos(t) & 0 \end{vmatrix} = \hat{k}$

d) From the magnitude of $\dfrac{d\hat{T}}{ds}$ in part c) we get $\kappa = \cos(t)$

e) $\dfrac{d\hat{B}}{ds} = (-\tau)\hat{N} = \dfrac{d\hat{B}}{dt}\dfrac{1}{|\underline{v}|} = \dfrac{d}{dt}(\hat{k}) \cdot \dfrac{1}{\sec(t)} = \underline{0}$

$\Rightarrow \tau = 0$!

or, you could recognize that the motion is in the $z=10$ plane, hence the binormal $\hat{B}$ will never change its orientation. Thus, $\tau = 0$.

4) f) $\underline{r}(t) = t\,\hat{i} - \ln(\cos(t))\,\hat{j} + 10\,\hat{k}$

$\underline{v}(t) = \hat{i} + \tan(t)\,\hat{j} + 0\,\hat{k}$

$\underline{a}(t) = 0\,\hat{i} + \sec^2(t)\,\hat{j} + 0\,\hat{k}$

Is $\underline{v} \perp \underline{a}$? $\quad \underline{v}\circ\underline{a} = 0 + \tan(t)\cdot\sec^2(t) + 0$

$= \sin(t)/\cos^3(t)$

This is equal to 0 only iff $t=0$

so at $t=0$, $\underline{v} \perp \underline{a}$

Is $\underline{v} \parallel \underline{a}$? No, because $\underline{v}(t)$ never has a zero $\hat{i}$ component,

or, you can show that $\underline{v}\times\underline{a} \neq \underline{0}$.

5) a) 6
b) 3
c) 8
d) 7