Be sure to include your name and a grading table on the front of your blue book. You must work all of the problems on this exam. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please clearly indicate the start of each new problem. Good luck!

1. (20 points) Two mice have nests at various positions in a giant bale of hay. Mike's nest is located at $A(2, 5, -6)$ and Kim's nest is located at $B(-4, 1, 6)$. One side of the bale of hay is the $x - z$ plane.

   (a) Kim's nest is located on a plane parallel to the side of the bale of hay designated as the $x - z$ plane. Find an equation for Kim's plane.

   (b) Mike's nest is located on a plane with a normal vector pointing from his nest to Kim's. Find an equation for the plane.

   (c) There is a string running straight through the bale of hay from Mike's nest along the line given by $\dfrac{(x-2)}{3} = \dfrac{(y-5)}{-1} = \dfrac{(z+6)}{2}$. There is a second string running straight through the bale of hay from Kim's nest along the line given by $\dfrac{(x+4)}{-4} = \dfrac{(y-1)}{2} = \dfrac{(z-6)}{4}$. Determine if these strings are parallel.

   (d) Determine if the strings in part c are perpendicular.

   (e) Mike and Kim would like to run from their respective nests along the strings to meet at the intersection of the strings. Determine if the strings intersect and if so, specify the point at which they intersect. If they do not intersect, be sure to state this.

2. (21 points) Consider the quadric surfaces defined by $Ax^2+By^2+Cz^2+Dx+Ey+Fz = G$, where the coefficients A through G only have the values $-1, 0$, or 1. For each of the surfaces described below, you need to assign values to the coefficients and then write out the simplest equation for the quadric surfaces.

   (a) Hyperboloid of two sheets centered on y-axis.

   (b) A right cylinder parallel to the x-axis.

   (c) A plane cutting a diagonal wedge across the first octant.

   (d) This question is unrelated to parts $a - c$. Sketch $x^2 - 16z^2 = 4y^2$ to scale in three dimensions using a right-handed coordinate system. Identify the shape, label the axes, and show the scale on each axis. Label at least one point on the surface not including the origin.

3. (20 points) Let $\mathbf{r}(t) = t^2\,\hat{\imath} + 3\,\hat{\jmath} + 2t\,\hat{k}$.

   (a) Find $\mathbf{v}(t)$

   (b) Find $\mathbf{a}(t)$

   (c) Find the tangential and normal *components* of acceleration at $t = 1$.

   (d) The binormal vector can be expressed as $\mathbf{B}(t) = \dfrac{\mathbf{v}(t) \times \mathbf{a}(t)}{|\mathbf{v}(t) \times \mathbf{a}(t)|}$.
   Using this definition, calculate $\mathbf{B}(t)$.

4. (21 points) Label each statement as true or false.

   (a) $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$

   (b) $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$.

   (c) $\mathbf{a} \times \mathbf{a} = |\mathbf{a}|^2$

   (d) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

   (e) $\mathbf{T} \cdot \mathbf{B} = 0$

   (f) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then $\mathbf{b} = \mathbf{c}$.

   (g) If $\mathbf{b} = \mathbf{c}$, then $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$.

5. (18 points) Some unrelated questions from Chapter 12.

   (a) Show that $z = e^{-t} \sin(\frac{x}{c})$ satisfies the heat equation. That is $\dfrac{\partial z}{\partial t} = c^2 \dfrac{\partial^2 z}{\partial x^2}$ where c is a constant and $c > 0$.

   (b) Suppose that three resistors are in parallel in an electrical circuit. If the resistances are R_1,R_2, and R_3 ohms, respectively, then the net resistance in the circuit is $R = \dfrac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}$ Compute **and** interpret $\dfrac{\partial R}{\partial R_1}$ (do not simplify $\dfrac{\partial R}{\partial R_1}$).

   (c) Let $f(x, y, z) = \ln(xyz^2)$. Find

      i. $\dfrac{\partial f}{\partial x}$

      ii. f_{xzy}

Projections, and distances from a point to a line and a plane

$$\text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

Arc length, Frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}|\,dt \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N} \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N} \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{[\,1+(f'(x))^2\,]^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}}$$

$$\tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T} \qquad a_T = \frac{d|\mathbf{v}|}{dt} \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$