1. $f = \text{volume} = xyz \qquad g = \text{constraint} = \frac{x}{a} + \frac{y}{b} + \frac{z}{c} - 1$

$$\nabla f = \lambda \nabla g$$

$$\left(yz\,\hat{\imath} + xz\,\hat{\jmath} + y\cancel{z}\,\hat{k} \right) = \lambda \left(\frac{\hat{\imath}}{a} + \frac{\hat{\jmath}}{b} + \frac{\hat{k}}{c} \right)$$

$$yz = \frac{\lambda}{a} \qquad xz = \frac{\lambda}{b} \qquad yx = \frac{\lambda}{c}$$

$$\lambda = yza = xzb \qquad \lambda = xzb = yxc$$

either $z = 0$ or $y = \frac{xb}{a}$ $\qquad$ either $x = 0$ or $z = \frac{xc}{a}$

(can't be this
it touches the
coor planes)

constraint: $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$$\frac{x}{a} + \frac{1}{b}\left(\frac{xb}{a}\right) + \frac{1}{c}\left(\frac{xc}{a}\right) = 1$$

$$\frac{3x}{a} = 1 \quad \Rightarrow \quad \boxed{x = \frac{a}{3}} \; \boxed{y = \frac{b}{3}} \; \boxed{z = \frac{c}{3}}$$

Point is $\boxed{\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)}$

2. $x^2 + 3y - y^3$

$f_x = 2x = 0 \Rightarrow x = 0$

$f_y = 3 - 3y^2 = 3(1-y^2) \Rightarrow y = \pm 1$

$f_{xx} = 2$

$f_{yy} = -2y$

$f_{xy} = 0$

point	$f_{xx}f_{yy} - f_{xy}^2$	f_{xx}	ID
$(0,1)$	$(2)(-2) = \text{neg}$	+	saddle
$(0,-1)$	$(2)(2) = \text{pos}$	+	local min
(a)			(b)

3.

a) $V = \int_{y=-3}^{3} \int_{x=-\sqrt{9-y^2}}^{+\sqrt{9-y^2}} +\sqrt{25-x^2-y^2}\, dx\, dy$

b) $V = \int_{\theta=0}^{2\pi} \int_{r=0}^{3} \sqrt{25-r^2}\, r\, dr\, d\theta$

c) $V = \int_{y=-3}^{3} \int_{x=-\sqrt{9-y^2}}^{+\sqrt{9-y^2}} \int_{z=0}^{\sqrt{25-x^2-y^2}} dz\, dx\, dy$

d) evaluate b

$$= \left[\int_{\theta=0}^{2\pi} d\theta\right]\left[\int_{r=0}^{3} \sqrt{25-r^2}\, r\, dr\right]$$

use $u = 25 - r^2$
$du = -2r\,dr$
$\frac{du}{-2} = r\,dr$

$$= (2\pi) \int \frac{\sqrt{u}}{-2}\, du = \frac{2\pi\, 2}{-2 \cdot 3} u^{3/2} \Big|$$

$$= -\frac{2\pi}{3} (25 - r^2)^{3/2} \Big|_{r=0}^{3} = -\frac{2\pi}{3}\left[(25-9)^{3/2} - (25-0)^{3/2}\right]$$

$$= -\frac{2\pi}{3}\left(16^{3/2} - 25^{3/2}\right) = -\frac{2\pi}{3}(16 \cdot 4 - 5 \cdot 25)$$

$$= -\frac{2\pi}{3}(64 - 125) = \frac{(61)(2\pi)}{3} = \frac{122\pi}{3}$$

4) a) $U(x,t) = f(x+ct) + g(x-ct)$

$r = x+ct \quad s = x-ct \quad r_x = 1 \quad s_x = 1$

$\frac{\partial r}{\partial t} = c \quad \frac{\partial s}{\partial t} = -c$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

~~u~~ $\left(f\left(u(r(x,t), s(x,t)\right)\right)$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial r}\overset{c}{\frac{\partial r}{\partial t}} + \frac{\partial u}{\partial s}\overset{-c}{\frac{\partial s}{\partial t}}$$

$$\frac{\partial u}{\partial t} = c\left[\frac{\partial u}{\partial r} - \frac{\partial u}{\partial s}\right]$$

$$\frac{\partial^2 u}{\partial t^2} = c\left[\frac{\partial^2 u}{\partial r^2}\overset{c}{\frac{\partial r}{\partial t}} + \frac{\partial^2 u}{\partial s^2}\overset{(+c)}{\frac{\partial s}{\partial t}}\right]$$

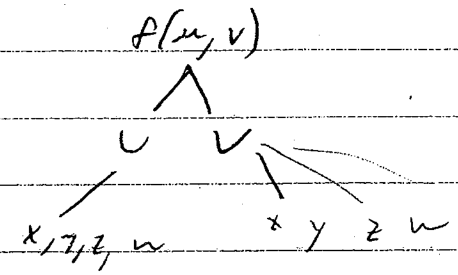
$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial s^2}\right)$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial r}\overset{1}{\frac{\partial r}{\partial x}} + \frac{\partial u}{\partial s}\overset{1}{\frac{\partial s}{\partial x}}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial s^2}$$

$$c^2[U_{rr} + U_{ss}] = c^2[U_{rr} + U_{ss}]$$

5)



$$\boxed{\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y}}$$

∂ ← partial cuz several independent variables

Let $u(x,y)$ and $v(x,y)$

c) $x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$x_r = \cos\theta \quad x_\theta = -r\sin\theta \qquad y_r = \sin\theta \quad y_\theta = r\cos\theta$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} \qquad \frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial r}$$

$$\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\frac{\partial y}{\partial \theta}$$

$$u_x\cos\theta + u_y\sin\theta = \frac{1}{\cancel{r}}\left[v_x(-\cancel{r}\sin\theta) + v_y(\cancel{r}\cos\theta)\right]$$

$$u_x\cos\theta + u_y\sin\theta = -v_x\sin\theta + v_y\cos\theta$$

but $u_x = v_y$ and $u_y = -v_x$ ☺

5) $f(x,y) = 106 - x^2 - y^2$ $(3,4)$

a) want $-\nabla f$

$\nabla f = -2x\hat{i} - 2y\hat{j}$ $\nabla f\Big|_{(3,4)} = -2(3)\hat{i} - 2(4)\hat{j} = -6\hat{i} - 8\hat{j}$

$-\nabla f = 6\hat{i} + 8\hat{j}$ direction need unit

$|\nabla f| = \sqrt{36+64} = \sqrt{100} = 10$

dir $= \dfrac{6\hat{i} + 8\hat{j}}{10}$ or $\boxed{.6\hat{i} + .8\hat{j}}$

b) $f(x,y) \approx f(x_0, y_0) + (x - x_0) f_x(x_0, y_0) + (y - y_0) f_y(x_0, y_0)$

$f(3,4) = 106 - 9 - 16 = 106 - 25 = 81 \Rightarrow z = 81$

$f_x(3,4) = -6$ $f_y(3,4) = -8$

$f(x,y) \approx 81 + (x-3)(-6) + (y-4)(-8)$

$81 - 6x + 18 - 8y + 32 = 131 - 6x - 8y$

c) $\displaystyle\int_0^1 \int_0^2 \int_0^{106-x^2-y^2} a \, dz\, dy\, dx$

$a \displaystyle\int_0^1 \int_0^2 106 - x^2 - y^2 \, dy\, dx$

$a \displaystyle\int_0^1 \left[106y - x^2 y - \frac{y^3}{3} \Big|_0^2 \right] dx$

$$a\int_0^1 \left[212 - 2x^2 - \frac{8}{3}\right] dx$$

$$a\left[212x - \frac{2x^3}{3} - \frac{8}{3}x\right|_0^1$$

$$a\left[212 - \frac{2}{3} - \frac{8}{3}\right] = a\left(212 - \frac{10}{3}\right)$$