APPM 2350 Final Exam Summer 2007

Be sure to include your name and a grading table on the front of your blue book. You must work all of the problems on this exam. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, talking horses, calculators, or electronic devices of any kind are NOT permitted. Note that this exam will be weighted as 150 points. Please start each problem on a new page. Good luck!

1. (22 points) Consider the following integral: $\displaystyle\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{x^2+y^2}} dz\,dy\,dx.$

   (a) Sketch the region that is being integrated.

   (b) Convert the original integral into cylindrical coordinates using the order $dr\,dz\,d\theta$ but **do not solve**.

   (c) Convert the original integral into spherical coordinates using the order $d\rho\,d\phi\,d\theta$ but **do not solve**.

   (d) Solve **one** of the integrals.

2. (22 points) Consider the velocity field $\mathbf{F}(x, y, z) = x^2\,\mathbf{i} + y^2\,\mathbf{j} + z^2\,\mathbf{k}$. Assume this field is flowing through a surface which is a cube bounded by the coordinate planes and the planes $x = a$, $y = a$, $z = a$.

   (a) Find the flux through the surface by direct calculation.

   (b) Verify the flux in (a) using a theorem from calc 3. Clearly state the theorem used.

   (c) Find circulation around the right surface panel by direct calculation.

   (d) Verify the circulation in (c) using a theorem from calc 3. Clearly state the theorem used.

3. (18 points) Consider the region R in the xy-plane bounded by $y = x + 1$, $y = x - 1$, $y = -x + 5$, and $y = -x + 3$.

   (a) Sketch the region.

   (b) Use the transformations $x = \frac{1}{2}(u + v)$ and $y = \frac{1}{2}(u - v)$ to sketch the new region S in the uv-plane.

   (c) If the density of the region R is given by $f(x, y) = e^{(x+y)}$, find the mass of the region by using the transformations given in (b).

4. (16 points) One can generate small amounts of finite improbability by hooking the logic circuits of a Bambleweeny 57 Sub-Meson Brain to an atomic vector plotter suspended in a strong Brownian Motion producer. The finite improbability field that is generated can be written as $\mathbf{F} = (y^2 + 4zx)\mathbf{i} + 2y(x + z)\mathbf{j} + (y^2 + 2x^2)\mathbf{k}$.

   (a) It turns out, that against almost all probability, **F** is conservative. Prove it.

   (b) It also turns out that the an **In**finite Improbability field is the potential of a finite improbability field. Find the potential of **F**.

   (c) For up to two bonus points, identify the source of inspiration for this question.

5. (22 points) Consider the curve $w = x^2y^3z^4$ where x, y, and z are defined in terms of the line $x = 2t + 1$, $y = 3t + 2$, and $z = 5t + 4$ for $t \geq 0$.

   (a) Find $\frac{dw}{dt}$.

   (b) What is the distance between the origin and the specified line?

   (c) The vector pointing parallel to the specified line is perpendicular to a plane that contains the origin. Find the equation for this plane.

   (d) What is the directional derivative of the curve w at the point $(1, -1, 1)$ in the direction of a vector pointing along the line?