INSTRUCTIONS: Computers, calculators, books, and crib sheets are not permitted. Write your (1) name, (2) instructor's name, and (3) lecture number (010, 020 or 030) on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

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1. (25 points) Consider the plane P defined by $2x + 2y + z = 2$ and another plane M that is parallel to P. Let A be a point on the plane M with the coordinates $(2, 2, 2)$.
   (a) Determine the standard equation of plane M.
   (b) How far is it from plane M to plane P?
   (c) Determine the parameterization of a line from point A to the closest point on plane P.
   (d) Determine the coordinates of the point on plane P closest to point A.

2. (25 points) Consider the movement of a particle traveling on the elliptic path defined by $\mathbf{r}(t) = \cos(t)\,\mathbf{i} + (\cos(t) - 1)\,\mathbf{j} + \sin(t)\,\mathbf{k}$ for $0 \le t \le 2\pi$.
   (a) Set up, *but do not evaluate*, the integral to determine the length of the path around the ellipse.
   (b) Calculate the acceleration, $\mathbf{a}(t)$, of a particle moving on the path described by $\mathbf{r}(t)$.
   (c) The path $\mathbf{r}(t)$ is actually the intersection of a cylinder and a plane. Determine the standard equation of the cylinder and the plane.
   (d) Show that the acceleration, $\mathbf{a}(t)$, is always in the plane described in part (c).
   (e) What is the torsion, τ, of the path $\mathbf{r}(t)$.

3. (25 points) Assume a particle moves along a path in three dimensional space defined by $\mathbf{r}(t) = t\,\mathbf{i} - \ln(\cos(t))\,\mathbf{j} + 10\,\mathbf{k}$, for $0 \le t \le \pi$.
   (a) The plane determined by the unit normal and binormal vectors, $\mathbf{N}$ and $\mathbf{B}$, at a point on the curve is called the local normal plane of the curve. At what point in space, P, is the local normal plane of $\mathbf{r}(t)$ parallel to the plane $x + y = 5$?
   (b) At the point P from part (a), compute the particle's velocity $\mathbf{v}$ and the unit tangent vector $\mathbf{T}$.
   (c) At the point P from part (a), determine the local unit normal vector $\mathbf{N}$.
   (d) At the point P from part (a), determine the local unit binormal vector $\mathbf{B}$.
   (e) Again, at the point P, determine the curvature κ.

4. (25 points) Consider the quadric surfaces defined by $Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz = G$, where the coefficients A through G only have the values -1, 0, or 1. For each of the surfaces described below, you need to assign values to the coefficients and then write out the simplest equation for the quadric surfaces.

   (a) A set of cones centered on the y-axis.

   (b) A single cone centered on the negative y-axis.

   (c) Hyperbolid of one sheet centered on the x-axis.

   (d) Hyperboloid of two sheets centered on the y-axis.

   (e) Paraboloid centered on the negative z-axis.

Projections, and distances from a point to a line and a plane

$$\text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

Arc length, Frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}|\,dt \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N} \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N} \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{[\,1+(f'(x))^2\,]^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}} \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T} \qquad a_T = \frac{d|\mathbf{v}|}{dt} \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$