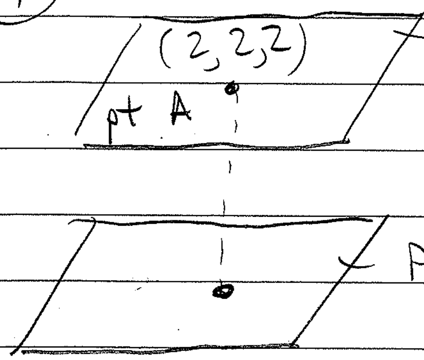
(1)



Plane M has same normal as plane P, so $\underline{n}_M = 2\hat{i} + 2\hat{j} + \hat{k}$

Plane P is $2x + 2y + z = 2$ with normal $\underline{n}_P = 2\hat{i} + 2\hat{j} + \hat{k}$

a) Since $\underline{n}_M$ for plane M is $\underline{n}_M = 2\hat{i} + 2\hat{j} + \hat{k}$ then its standard equation must be

$$2x + 2y + z = D$$

Evaluating this at pt $(2,2,2)$ we get

$$2x + 2y + z = 10 \text{ for plane M.}$$

b) We need the dist from $A(2,2,2)$ to plane P which, by the way, contains the point $B(0,0,2)$. Then vector $\underline{BA} = 2\hat{i} + 2\hat{j} + 0\hat{k}$. Finally the distance is

$$d = \left| BA \circ \frac{\underline{n}}{|\underline{n}|} \right|$$
$$= \left| (2\hat{i} + 2\hat{j} + 0\hat{k}) \circ (2\hat{i} + 2\hat{j} + \hat{k}) \cdot \frac{1}{3} \right|$$
$$= 8/3$$

c) The line through pt $A(2,2,2)$ in direction of $\underline{n} = 2\hat{i} + 2\hat{j} + 1\hat{k}$ is

$$\underline{v}(t) = \underline{OA} + t\underline{n} = (2\hat{i} + 2\hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} + \hat{k})$$
$$= \underbrace{(2+2t)}_{x(t)}\hat{i} + \underbrace{(2+2t)}_{y(t)}\hat{j} + \underbrace{(2+t)}_{z(t)}\hat{k}$$

1 cont.

d) We need a point on the line $\vec{r}(t)$ that satisfy the std. eqn for plane P. Then

$x(t)$, $y(t)$ and $z(t)$ must satisfy

$$2x + 2y + z = 2$$

or

$$2(2+2t) + 2(2+2t) + 1.(2+t) = 2$$

or

$$t = -8/9$$

Thus the point is

$$\vec{r}(t = -8/9) = \left[2 + 2\left(\tfrac{-8}{9}\right)\right]\hat{i} + \left[2 + 2\left(\tfrac{-8}{9}\right)\right]\hat{j} + \left[2 + (-8/9)\right]\hat{k}$$

$$= 2/9\,\hat{i} + 2/9\,\hat{j} + 10/9\,\hat{k}$$

So the point is

$$\left(\tfrac{2}{9}, \tfrac{2}{9}, \tfrac{10}{9}\right) \leftarrow$$

(2) $\underline{r}(t) = \cos(t)\,\hat{i} + (\cos(t) - 1)\,\hat{j} + \sin(t)\,\hat{k} \qquad 0 \le t \le 2\pi$

a) $\underline{v} = -\sin t\,\hat{i} - \sin t\,\hat{j} + \cos t\,\hat{k}$

$|\underline{v}| = (\sin^2 t + \sin^2 t + \cos^2 t)^{1/2}$

$= (1 + \sin^2 t)^{1/2}$

thus

$$s = \int_{t=0}^{2\pi} |\underline{v}|\,dt = \int_{t=0}^{2\pi} \sqrt{1 + \sin^2 t}\,dt$$

b) $\underline{a} = -\cos t\,\hat{i} - \cos t\,\hat{j} - \sin t\,\hat{k}$

c) From the $\hat{i}$ + $\hat{k}$ components of $\underline{r}(t)$, the cylinder must be $x^2 + z^2 = 1$

To find the plane, you could take 3 pts on the path. For example

$\underline{r}(t=0) = \hat{i} + 0\hat{j} + 0\hat{k} \qquad (1,0,0)$

$\underline{r}(t=\pi/2) = 0\hat{i} - \hat{j} + \hat{k} \qquad (0,-1,1)$

$\underline{r}(t=\pi) = -\hat{i} - 2\hat{j} + 0\hat{k} \qquad (-1,-2,0)$

Using these three points, the plane can be found to be $x - y = 1$.

You might also note from $\underline{r}(t) = \cos t\,\hat{i} + (\cos t - 1)\hat{j} + \sin t\,\hat{k}$ that $y = x - 1$ for all points on the path.

Hence $x - y = 1$

(2) con't.

d) Plane in part c) has normal $\underline{n} = \hat{i} - \hat{j}$

So if $\underline{a}$ is in the plane, we would expect

$$\underline{a} \circ \underline{n} = 0 \quad \text{(ie } \underline{a} + \underline{n} \text{ are orthogonal)}$$

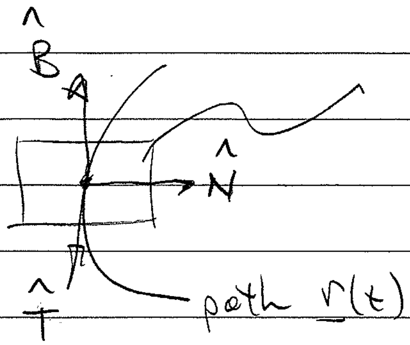
$$(-\cos t\,\hat{i} - \cos t\,\hat{j} - \sin t\,\hat{k}) \circ (\hat{i} - \hat{j})$$

$$= -\cos t + \cos t = 0$$

$\therefore$ $\underline{a}$ is in plane $x - y = 1$

e) The path $\underline{r}(t)$ is in a plane and hence the torsion must be 0!

(3) $\underline{r}(t) = t\,\hat{\imath} - \ln(\cos t)\,\hat{\jmath} + 10\,\hat{k}$ for $0 \le t \le \pi$



local normal plane formed by vectors $\hat{N}$ & $\hat{B}$

a) The local normal plane has unit normal $\hat{T}$ to path $\underline{r}(t)$. Thus $\underline{V}(t)$ is $\parallel$ to $\hat{T}$ and so we want to know, when is $\underline{V}(t) \parallel$ to the normal to the plane $x+y=5$?

Ie when is

$$\underline{V} = \hat{\imath} + \tan t\,\hat{\jmath} \quad \parallel \text{ to } \quad \underline{n} = \hat{\imath} + \hat{\jmath}.$$

This happens when $\tan t = 1$, so $t = \pi/4$

This is a point

$$\underline{r}(t=\tfrac{\pi}{4}) = \tfrac{\pi}{4}\,\hat{\imath} - \ln(1/\sqrt{2})\,\hat{\jmath} + 10\,\hat{k}$$

$$\text{or } (\pi/4,\ -\ln(1/\sqrt{2}),\ 10) \leftarrow$$

b) $\underline{V}(t=\pi/4) = \hat{\imath} + \tan\frac{\pi}{4}\,\hat{\jmath} = \hat{\imath} + \hat{\jmath}$ $\leftarrow$

so $\hat{t} = \underline{V}/|\underline{V}| = \frac{1}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})$ $\leftarrow$

c) $$\frac{d\hat{T}}{ds} = \frac{d\hat{t}}{dt}\cdot\frac{1}{|\underline{V}|} = \kappa \cdot \hat{N}$$

unit vector in dir of $\frac{d\hat{t}}{ds}$

mag. of vector $d\hat{t}/ds$

$$\underline{V} = \hat{\imath} + \tan t\,\hat{\jmath} \quad \text{so} \quad |\underline{V}| = \sqrt{1+\tan^2 t} = \sec t$$

thus

$$\hat{T} = \frac{\underline{V}}{|\underline{V}|} = \frac{1}{\sec t}(\hat{\imath} + \tan t\,\hat{\jmath}) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath}$$

(3) cont

So $\frac{d\hat{T}}{ds} = \frac{d\hat{T}}{dt}\cdot\frac{1}{|\underline{v}|} = (-\sin t\,\hat{i} + \cos t\,\hat{j})\cdot\cot$

Thus $\hat{N} = \frac{d\hat{T}/ds}{|d\hat{T}/ds|} = \frac{d\hat{T}/dt}{|d\hat{T}/dt|} = \frac{-\sin t\,\hat{i} + \cos t\,\hat{j}}{1}$

$= -\sin t\,\hat{i} + \cos t\,\hat{j}$

In particular $\hat{N}\Big|_{t=\pi/4} = -\frac{1}{\sqrt{2}}\,\hat{i} + \frac{1}{\sqrt{2}}\,\hat{j}$

d) $\hat{B} = \hat{T}\times\hat{N} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \end{vmatrix} = \hat{k}(\cos^2 t + \sin^2 t)$
$= \hat{k}$

so $\hat{B}\Big|_{t=\pi/4} = \hat{k}$

e) $\kappa = \left|\frac{d\hat{T}}{ds}\right| = \left|\frac{d\hat{T}}{dt}\cdot\frac{1}{|\underline{v}|}\right| = \frac{1}{|\underline{v}|}\cdot\left|\frac{d\hat{T}}{dt}\right|$

$= |\cot|\cdot|-\sin t\,\hat{i} + \cos t\,\hat{j}| = |\cot|$

$\kappa\Big|_{t=\pi/4} = 1/\sqrt{2}$

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a) $x^2 - y^2 + z^2 = 0$

b) $y = -\sqrt{x^2 + z^2}$

c) $-x^2 + y^2 + z^2 = 1$

d) $x^2 - y^2 + z^2 = -1$

e) $x^2 + y^2 + z = 0$