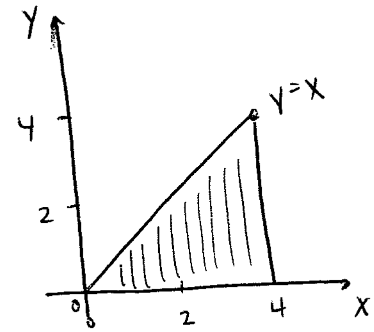
1. $f(x,y) = 20 - 16x - 4y + 4x^2 + y^2$

For critical pt, set gradient = 0



$\nabla f = \langle 8x-16,\ 2y-4 \rangle$

$x = 2,\ y = 2$

↳ critical pt at $(2,2)$

Check boundaries

$x=4 \Rightarrow f(4,y) = 20 - 4y + y^2$

$f'(4,y) = -4 + 2y = 0 \Rightarrow y = 2$

pt $(4,2)$

$y=x \Rightarrow f(x,x) = 20 - 20x + 5x^2$

$f'(x,x) = -20 + 10x = 0 \Rightarrow x = 2$

pt $(2,2)$

$y=0 \Rightarrow f(x,0) = 20 - 16x + 4x^2$

$f'(x,0) = -16 + 8x = 0 \Rightarrow x = 2$

pt $(2,0)$

Check endpoints

$(0,0)$

$(4,0)$

$(4,4)$

Extrema

@ $(0,0)$, $f = 20$

@ $(4,0)$, $f = 20$

@ $(4,4)$, $f = 20$

@ $(2,2)$, $f = 0$

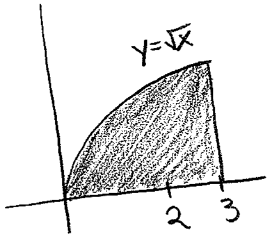
@ $(4,2)$, $f = 16$

@ $(2,0)$, $f = 4$

absolute max @ $(0,0)$, $(4,0)$, $(4,4)$

absolute min @ $(2,2)$

(2)



distance $d=\sqrt{(x-2)^2+y^2}$

$$d^2=(x-2)^2+y^2=f$$

$$y-\sqrt{x}=g$$

$$\nabla f=\lambda\nabla g$$

$$\langle 2(x-2),\ 2y\rangle=\lambda\left\langle \frac{-1}{2\sqrt{x}},\ 1\right\rangle$$

$$\boxed{\begin{aligned} &(1)\quad 2x-4=\frac{-\lambda}{2\sqrt{x}} \\ &(2)\quad 2y=\lambda \\ &(3)\quad y=\sqrt{x} \end{aligned}}$$

plug in y for $\sqrt{x}$

$$2x-4=\frac{-\lambda}{2y}$$

plug in $\lambda=2y$

$$2x-4=\frac{-\lambda}{\lambda}$$

$$2x-4=-1$$

$$2x=3$$

$$x=3/2$$

$$y=\sqrt{3/2}$$

Closest point:

$$\boxed{\left(3/2,\ \sqrt{3/2}\right)}$$

furthest point

check end points

$$d=\sqrt{(x-2)^2+y^2}\ \Big|_{(0,0)}$$

$$d=\sqrt{(-2)^2+0^2}$$

$$d=2$$

$$d=\sqrt{(x-2)^2+y^2}\ \Big|_{(3,\sqrt{3})}$$

$$d=\sqrt{(3-2)^2+(\sqrt{3})^2}$$

$$d=\sqrt{1+3}$$

$$d=\sqrt{4}=2$$

furthest points:

$$\boxed{(0,0)\ \ \&\ \ (3,\sqrt{3})}$$

3 $\vec{r} = 2i + j + k$ $\quad T(2,1,1) = 5$

$\vec{v} = 2i + 2j + k$ $\quad \nabla T(2,1,1) = 3i + 2j + k$

$\vec{a} = 2j$

a). $\nabla T \cdot \vec{v} = 3(2) + 2(2) + 1(1) = \boxed{11}$

if it was with respect to S, use $\nabla T \cdot \vec{v}$ but with respect to time do not divide by $|\vec{v}|$

b). $\underset{\substack{\uparrow \\ \text{from a}}}{11}\,(\underset{\substack{\uparrow \\ \Delta t}}{.1})$

c). $K = \dfrac{|\vec{v} \times \vec{a}|}{|\vec{v}^3|}$ $\qquad \vec{v} \times \vec{a} = \begin{vmatrix} i & (-)j & k \\ 2 & 2 & 1 \\ 0 & 2 & 0 \end{vmatrix} = -2i + 4k$

$|\vec{v} \times \vec{a}| = \sqrt{20} = 2\sqrt{5}$

$|\vec{v}| = 3$

$$\frac{|\vec{v} \times \vec{a}|}{|\vec{v}^3|} = \frac{2\sqrt{5}}{27}$$

D $\tau = -\dfrac{dB}{ds} \cdot N$ $\qquad B = T \times N$

$T = \dfrac{v}{|v|}$ $\qquad N = \dfrac{a}{|a|}$

This will give B at the point (2,1,1) however we can not calculate torsion because we can not calculate $\dfrac{dB}{ds}$

4 a) $\frac{10}{x} = \frac{20}{x^2}$ $\quad 10x = 20 \quad x = 2$

A: $\frac{10}{x}$ $\qquad$ C: $\frac{20}{x^2}$ $\qquad$ 4 boundary lines

B: $\frac{20}{x}$ $\qquad$ D: $\frac{40}{x^2}$

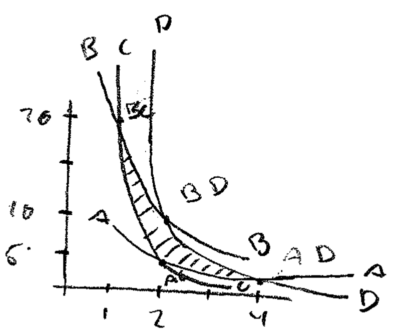
<u>intersections</u>

AC: $\frac{10}{x} = \frac{20}{x^2}$ $\quad x = 2 \quad y = 5$

BC: $\frac{20}{x} = \frac{20}{x^2}$ $\quad x = 1 \quad y = 20$

AD: $\frac{10}{x} = \frac{40}{x^2}$ $\quad x = 4 \quad y = 2.5$

BD: $\frac{20}{x} = \frac{40}{x^2}$ $\quad x = 2 \quad y = 10$

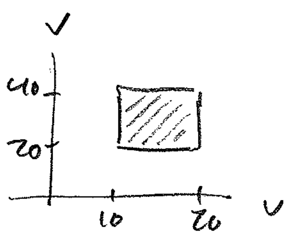


Draw intersection pts & connect lines
↳ every line has two pts on graph

Hard to draw

b) $u = xy \qquad v = x^2 y \qquad 10 \le u \le 20$

$20 \le v \le 40$



c) $y = \frac{u}{x}$

$v = x^2\left(\frac{u}{x}\right)$

$v = xu$

$x = \frac{v}{u}$ $\qquad e^{xy} = e^{u}$

$u = \frac{v}{u} y$

$\frac{u^2}{v} = y$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} -\frac{v}{u^2} & \frac{1}{u} \\ \frac{2u}{v} & \frac{-u^2}{v^2} \end{vmatrix} = \frac{vu^2}{v^2u^2} - \frac{2u}{v} \Rightarrow \frac{1}{v} - \frac{2}{v} = \left|-\frac{1}{v}\right| = \frac{1}{v}$$

$$\int_{20}^{40}\int_{10}^{20} \frac{e^u}{v}\, du\, dv$$

$$= (e^{20} - e^{10}) \int_{20}^{40} \frac{1}{v}\, dv$$

$$\boxed{= (e^{20} - e^{10})(\ln 40 - \ln 20)}$$

⑤ $\vec{F} = 6x^2y\hat{\imath} + (2x^3 + 2yz)\hat{\jmath} + y^2\hat{k}$

$\vec{r}(t) = t^3\hat{\imath} + 2\hat{\jmath} + t^2\hat{k} \quad 0 \le t \le 1$

a) $\int \vec{F}(\vec{r}(t)) \cdot \vec{T}\, ds$ (line integral) $= \int_a^b F \cdot \frac{r'(t)}{\|r'(t)\|} \cdot \|r'(t)\|\, dt$

$= \int_0^1 \langle 6(t^3)^2(2),\ 2(t^3)^3 + 2(2)(t^2),\ 4 \rangle \cdot \langle 3t^2, 0, 2t \rangle\, dt$

$= \int_0^1 36t^8 + 8t\, dt = 4t^9 + 4t^2 \Big|_0^1 = 8$

b) Check if function is conservative.

$\nabla \times F = 0$ or $\int 6x^2y\, dx = 2x^3y + g(y,z)$

$(2x^3y + g(y,z))\frac{\partial}{\partial y} = 2x^3 + 2yz$

$= 2x^3 + \frac{\partial g}{\partial y} = 2x^3 + 2yz \quad \therefore \frac{\partial g}{\partial y} = 2yz$

$g(y,z) = y^2z + h(z)$

$\left(\frac{\partial}{\partial z}\left(2x^3y + y^2z + h(z)\right)\right) = y^2 + h'(z) = y^2 \quad \therefore h'(z) = 0$

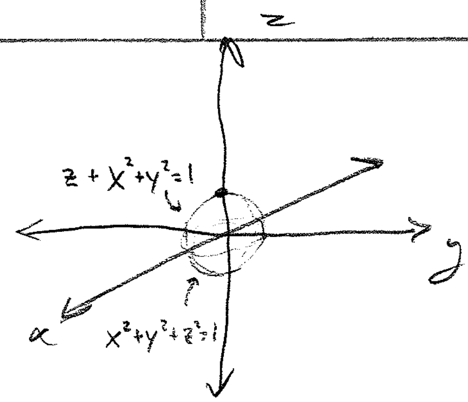
$f = 2x^3y + y^2z$ $\quad P_1 = \langle 0,2,0 \rangle$ $\quad P_2 = \langle 1,2,1 \rangle$ * Plug in $t=0$ and $t=1$ to $\vec{r}(t)$

$f(1,2,1) - f(0,2,0) = 8$

ⓒ Calculate Flux using line integral

ⓓ Bummer.

6) a)



$x^2+y^2=1$

$$\boxed{\vec{r}(t) = \langle \cos t, \sin t \rangle \quad 0 \le t \le 2\pi}$$

b) $\int_c F \cdot T \, ds$ $\qquad F = \langle 3y, -3x \rangle$

$F(\vec{r}(t))$ $\qquad \vec{v}(t)$

By definition of circulation $\Rightarrow$ $\int_0^{2\pi} \langle 3\sin t, -3\cos t \rangle \cdot \langle -\sin t, \cos t \rangle \, dt$

$\int_0^{2\pi} -3\sin^2 t - 3\cos^2 t \; dt$

$-3 \int_0^{2\pi} (\sin^2 t + \cos^2 t) \, dt$

$-3 [2\pi] = \boxed{-6\pi}$

c) <u>Greens Thm</u>

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y & -3x & 0 \end{vmatrix}$$

$$(0-0)\hat{\imath} + (0-0)\hat{\jmath} + (-3-3)\hat{k}$$

$$-6\hat{k}$$

$$\nabla \times F = \langle 0, 0, -6 \rangle$$

$$\nabla \times F \cdot \hat{k} = -6$$

$$\int_0^{2\pi}\int_0^1 -6 \; r\,dr\,d\theta$$

$$-6 \int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^1 d\theta$$

$$-3 \int_0^{2\pi} d\theta = \boxed{-6\pi}$$

<u>Stokes Thm</u>

$$\nabla \times F = \langle 0, 0, -6 \rangle$$

$$\iint_{R_{xy}} \nabla \times F \cdot \overbrace{\frac{\nabla g}{\cancel{|\nabla g|}}}^{n} \overbrace{\left(\frac{\cancel{|\nabla g|}}{|\nabla g \cdot p|}\right)}^{d\sigma} dA$$

$$\langle 0, 0, -6 \rangle \cdot \langle 2x, 2y, 1 \rangle \, dA$$

$$\iint_{R_{xy}} -6 \quad dA$$

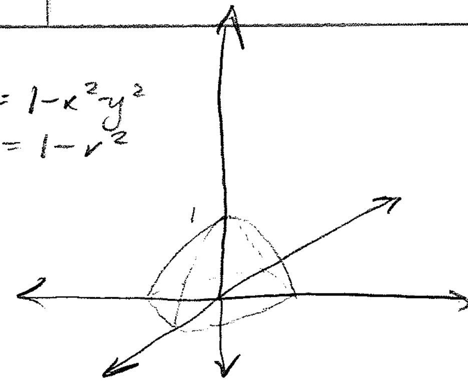
$$\int_0^{2\pi}\int_0^1 -6 \; r\,dr\,d\theta = \boxed{-6\pi}$$

d)

Divergence Theorem $= \iiint_{D_{xyz}} 4 \quad dx\,dy\,dz$

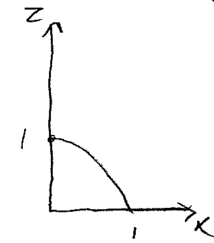
$z = 1-x^2-y^2$

$z = 1-r^2$



$$\int_0^1\int_0^{2\pi}\int_0^{1-r^2} 4 \quad r\,dz\,d\theta\,dr$$

$\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle \cdot \langle 2x, 2y, 0 \rangle = \underline{4}$



$$4\int_0^1 r \int_0^{2\pi} \left[z \right]_0^{1-r^2} d\theta\,dr$$

$$= 4\int_0^1 r\int_0^{2\pi} 1-r^2 \; d\theta\; dr$$

$$= 4\int_0^1 (r-r^3) \left[\theta\right]_0^{2\pi} dr$$

$$= 8\pi \int_0^1 r-r^3 \; dr \quad = \quad 8\pi \left[\frac{r^2}{2} - \frac{r^4}{4}\right]_0^1$$

$$= 8\pi\left(\frac{1}{2}-\frac{1}{4}\right) = \boxed{2\pi}$$

e) $\iint_S \vec{G}\cdot\vec{n}\; d\sigma$ (Definition)

It works. $\boxed{= 2\pi}$