APPM 2350 **EXAM 1** **SPRING 2008**

INSTRUCTIONS: Electronic devices are not permitted in the exam. Write your name, your instructor's name, and your recitation number on the front of your bluebook. Start each problem on a new right-hand page. Justify your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

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1. (20 points) For each of the following unrelated questions, answer either ALWAYS TRUE or NOT ALWAYS TRUE. No justification is necessary.
   (a) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$.
   (b) If $\mathbf{A} \cdot \mathbf{B} = \mathbf{C} \cdot \mathbf{B}$ and $\mathbf{B} \neq \mathbf{0}$, then $\mathbf{A} = \mathbf{B}$.
   (c) $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{A} = (\mathbf{B} \times \mathbf{A}) \cdot \mathbf{A}$.
   (d) If an object moves on a circular path, its acceleration vector is constant.
   (e) If an object moves with a constant speed, then the acceleration vector must be perpendicular to the velocity vector.

2. (30 points) Consider a plane P_1 that intersects the principle axes at the locations $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1/2)$. Furthermore, consider a second plane P_2 parallel to P_1 that intersects the principle axes twice as far away from the origin as P_1. For example, P_2 intersects the x-axis at the point $(2, 0, 0)$, and so on.
   (a) Determine the standard equation of plane P_1 and its unit normal vector.
   (b) Determine the parameterization of a line, L, that passes through the origin and is perpendicular to plane P_1.
   (c) What are the coordinates of the point, A, where the line intersects the plane P_1?
   (d) Now determine the standard equation of the second plane P_2.
   (e) What are the coordinates of the point, B, where the line intersects the plane P_2?
   (f) Being a clever Calculus III student, you realize that you can determine the distance between planes P_1 and P_2 by performing an arc length calculation along the path $\mathbf{r}(t)$ describing the line L from the point A to B. What is this distance?

3. (30 points) Over a large portion of a ski slope, the surface is essentially a plane that passes through the points $P_1(2, 0, 1)$, $P_2(0, 0, 1)$, and $P_3(0, 5, 0)$ where all distances are measured in kilometers. Kate goes down the slope such that her x and y coordinates are $x(t) = 2 - 2t$ and $y(t) = 5 \sin\left(\frac{\pi t}{2}\right)$. Kate starts her run at time $t = 0$ at point P_1 and ends her run at P_3.
   (a) Determine the standard equation of the plane representing the ski slope.
   (b) Determine the z-coordinate of Kate's path, $z(t)$, then clearly write out the full paramaterization of her path $\mathbf{r}(t)$ in 3-D.
   (c) At what time is Kate at the bottom of the path?
   (d) Is Kate's velocity ever parallel to her acceleration? If so, when?
   (e) Is Kate's velocity ever perdendicular to her acceleration? If so, when?
   (f) What is the minimum value of the curvature, κ, on Kate's path, and when does it occur?

4. (20 points) Name (ellipsoid, elliptic paraboloid, hyperbolic paraboloid, etc.) each of the following surfaces.

   (a) $4x^2 + 4y^2 + z^2 = 16$.

   (b) $x^2 + y^2 - z^2 = 1$.

   (c) $z^2 - x^2 - y^2 = 1$.

   (d) $y^2 + z^2 = x^2$.

   (e) $y^2 - x^2 = z$.

Projections, and distances from a point to a line and a plane

$$\text{proj}_{\mathbf{A}}\mathbf{B} = \left(\frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{A}\cdot\mathbf{A}}\right)\mathbf{A} \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

Arc length, Frenet formulas, and tangential and normal acceleration components

$$ds = |\mathbf{v}|\,dt \qquad \mathbf{T} = \frac{d\mathbf{r}}{ds} = \frac{\mathbf{v}}{|\mathbf{v}|} \qquad \mathbf{N} = \frac{d\mathbf{T}/ds}{|d\mathbf{T}/ds|} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N} \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N} \qquad \kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{v}\times\mathbf{a}|}{|\mathbf{v}|^3} = \frac{|f''(x)|}{[\,1+(f'(x))^2\,]^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}} \qquad \tau = -\frac{d\mathbf{B}}{ds}\cdot\mathbf{N}$$

$$\mathbf{a} = a_N\mathbf{N} + a_T\mathbf{T} \qquad a_T = \frac{d|\mathbf{v}|}{dt} \qquad a_N = \kappa|\mathbf{v}|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}$$