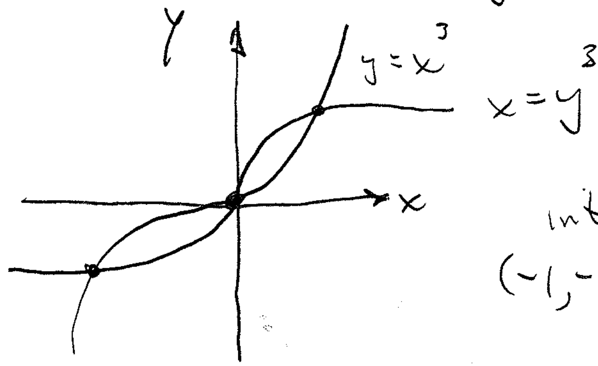
(1) $f = 4xy - x^4 - y^4 + 100$

$f_x = 4y - 4x^3 = 0$ $\qquad$ $f_y = 4x - 4y^3 = 0$



intersections are at $(-1,-1)$ $(0,0)$ + $(1,1)$

$f_{xx} = -12x^2$ $\qquad$ $f_{yy} = -12y^2$ $\qquad$ $f_{xy} = 4$

$D = 144x^2y^2 - 16$ $\qquad$ $D\big|_{-1,-1} = 128 > 0$ and $f_{xx} = -12$

so max

$D\big|_{0,0} = -16 < 0$ so sad. pt.

$D\big|_{1,1} = 128 > 0$ and $f_{xx} = -12$

so max

Molly should look at $(-1,-1)$ and $(1,1)$

(2)

a) $\frac{dg}{ds} = \nabla g \circ \hat{u} = \nabla g \circ \frac{\underline{v}}{|\underline{v}|} = (4\hat{i} + 0\hat{j} + 3\hat{k}) \circ (\hat{i} + 2\hat{j} + 3\hat{k}) \frac{1}{\sqrt{14}}$

$= 13/\sqrt{14}$

b) $\frac{dg}{dt} = \frac{dg}{ds}\frac{ds}{dt} = \nabla g \circ \frac{v}{|\underline{v}|} \cdot |\underline{v}| = \nabla g \circ \underline{v} = 13$

c) $\Delta g \approx \frac{dg}{dt} \cdot \Delta t = 13 \cdot \frac{1}{10} = 1.3$

d) $\frac{dg}{ds} = \nabla g \circ \hat{u} = \nabla g \circ \frac{\nabla g}{|\nabla g|} = |\nabla g| = 5$

$\Delta g \approx \frac{dg}{ds} \Delta s = 5 \times 0.2 = 1$

e) $k = \frac{|\underline{v} \times \underline{a}|}{|\underline{v}|^3}$

$\underline{v} \times \underline{a} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 2 & 6 \end{vmatrix} = \hat{i}\,6 - \hat{j}\,0 + \hat{k}(-2)$

$k = \frac{\sqrt{40}}{(\sqrt{14})^3}$

$|\underline{v} \times \underline{a}| = \sqrt{40}$

$|\underline{v}| = \sqrt{14}$

(3) $f = A(w,H) = w \cdot H$ (obj) $\quad g = 2w + 2H = P$ const.

$$\nabla f = \lambda \nabla g$$

$$H\hat{i} + w\hat{j} = \lambda(2\hat{i} + 2\hat{j})$$

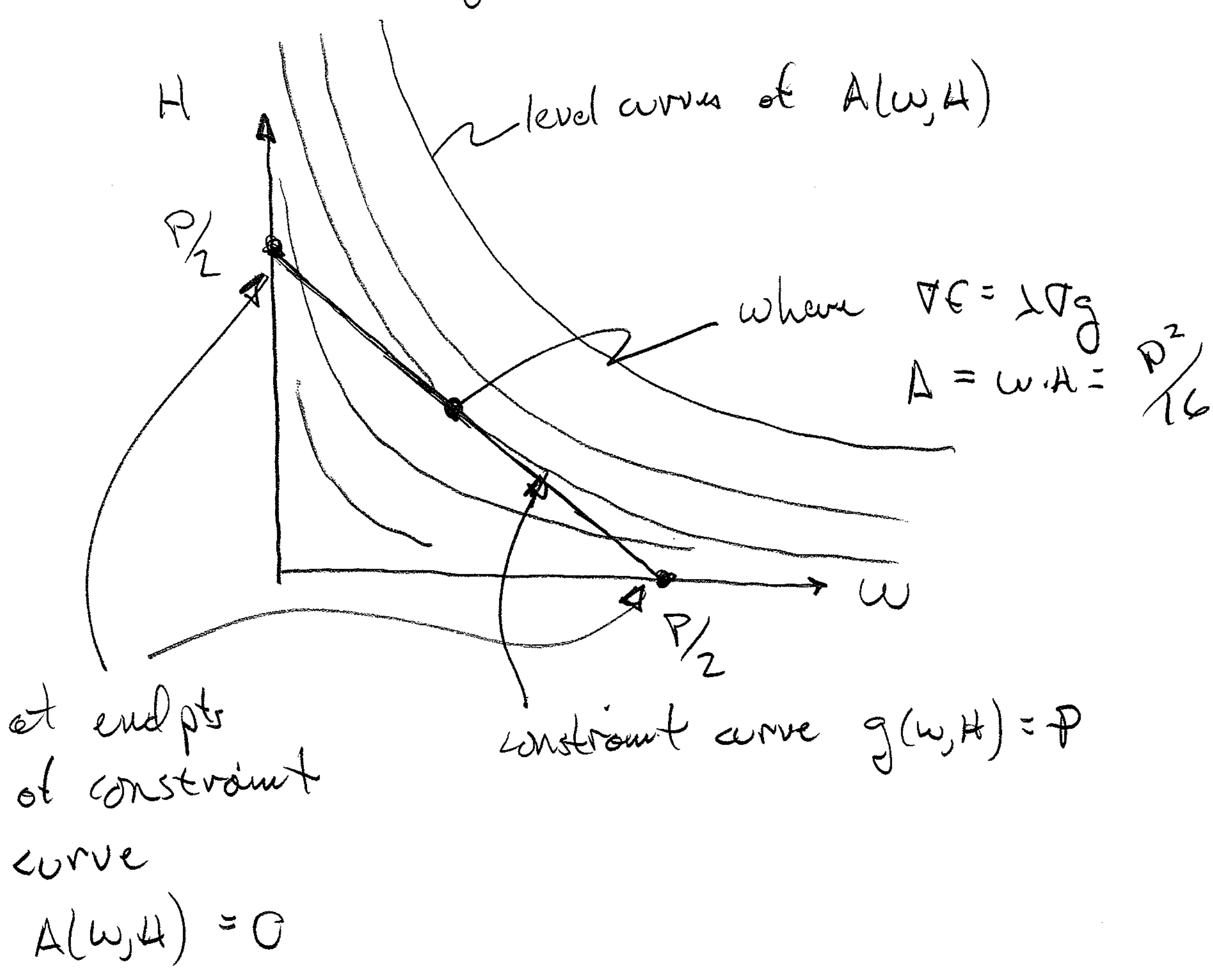
(1) $H = 2\lambda$ $\qquad$ (2) $w = 2\lambda$ $\qquad$ (3) $2w + 2H = P$

$4\lambda + 4\lambda = P$

$\lambda = P/8$

so $H = 2\lambda = P/4$ and $w = 2\lambda = P/4$

so $H = w = P/4$ (square!)



(4) $$f = x^2 + y^2 + \cos(xy)$$

$f_x = 2x - y\sin(xy)$ $\qquad$ $f_y = 2y - x\sin(xy)$

$f_{xx} = 2 - y^2\cos(xy)$ $\qquad$ $f_{yy} = 2 - x^2\cos(xy)$

$$f_{xy} = -\sin(xy) - xy\cos(xy)$$

a) $f \approx f(0,0) + \cancel{f_x(0,0)}^{0} x + \cancel{f_y(0,0)}^{0} y$

$= 1$

b) bounds on: f_{xx} use $2 + 0.1^2 = 2.01$

f_{yy} use $2 + 0.1^2 = 2.01$

f_{xy} use $1 + 0.1^2 = 1.01$

so

$$|\text{error}| \le \frac{2.01}{2}\left[\,\left|\tfrac{1}{10}\right| + \left|\tfrac{1}{10}\right|\,\right]^2 = \frac{2.01}{2}(0.2)^2 = 0.0402$$

c) $f(0.1, 0.1) = (0.1)^2 + (0.1)^2 + \cos(0.1 \times 0.1)$

$= 0.02 + 0.99995$

$= 1.01995$ "exact"

4

d) $f(0.1, 0.1) \approx 1$ approx

c) actual error $= 0.01995$ (is indeed smaller than est. of error)

f_{exact}	f_{linear}	error bound	actual error
1.01995	1	0.0402	0.01995