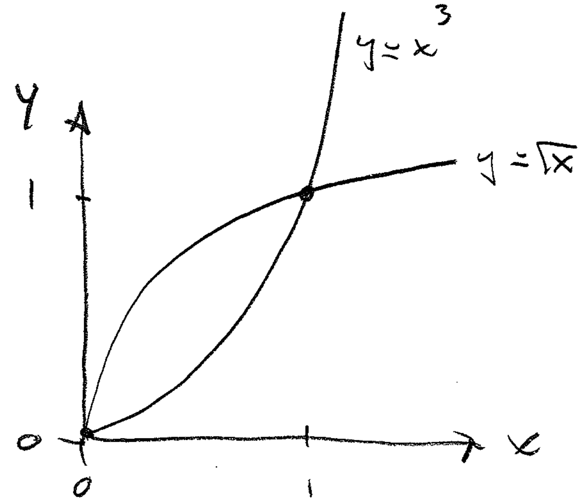
① $$A = \int_{y=0}^{1} \int_{x=y^2}^{\sqrt[3]{y}} dx\,dy$$

a)



b) $$A = \int_{x=0}^{1} \int_{y=x^3}^{\sqrt{x}} dy\,dx$$

c) $$A = \int_{x=0}^{1} \int_{y=x^3}^{\sqrt{x}} dy\,dx \quad = \int_{x=0}^{1} \left(\sqrt{x} - x^3\right) dx$$

$$= \left(\frac{2}{3} x^{3/2} - \frac{x^4}{4} \right|_{x=0}^{1} = \frac{2}{3} - \frac{1}{4} = 5/12 \longleftarrow$$

(2)

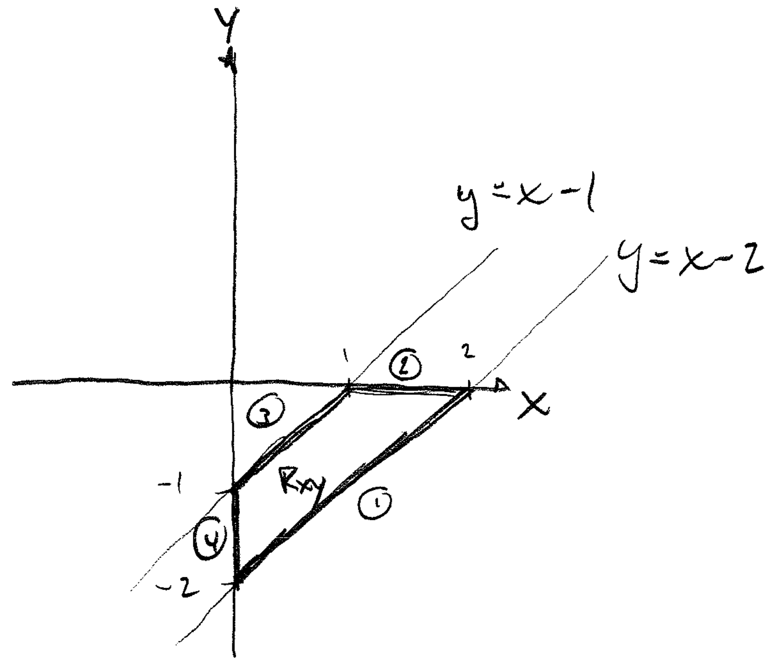
a) $u = x+y$

$v = x-y$

gives

$$x = \frac{1}{2}(u+v)$$

$$y = \frac{1}{2}(u-v)$$



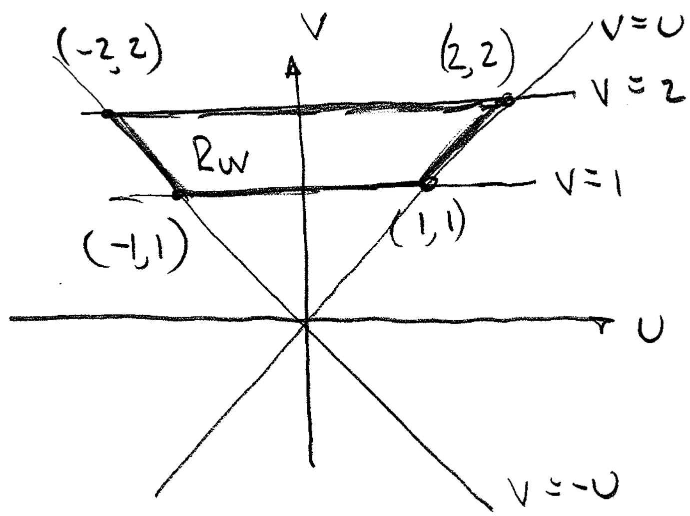
b) $J(u,v) = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$

(1) $y - x = -2$
becomes $v = 2$

(2) $y = 0$
becomes $u = v$

(3) $y - x = -1$
becomes $v = 1$

(4) $x = 0$
becomes $v = -u$



(2) cont'

c) $c = \iint_{R_{xy}} e^{(x+y)/(x-y)}\,dx\,dy$

$$= \int_{v=1}^{2} \int_{u=-v}^{u=v} e^{u/v} \left| \frac{-1}{2} \right| du\,dv$$

d) $$c = \frac{1}{2} \int_{v=1}^{2} \left(v e^{u/v} \Big|_{u=-v}^{v} \right) dv$$

$$= \frac{1}{2} \int_{v=1}^{2} v(e - e^{-1})\,dv$$

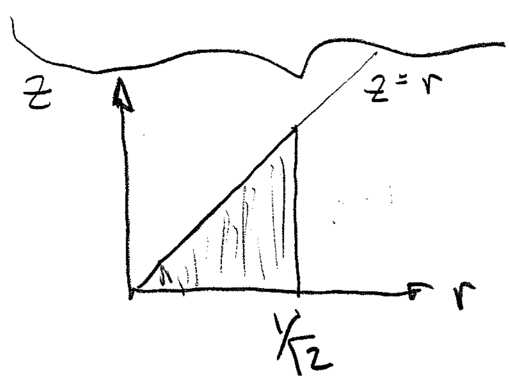
$$= \frac{1}{2}(e - e^{-1}) \frac{v^2}{2} \Big|_{v=1}^{2}$$

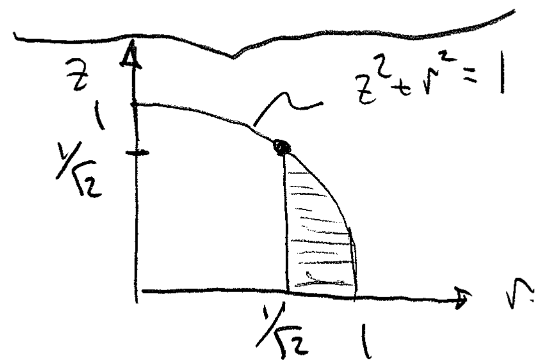
$$= 3(e - e^{-1})$$

(3)

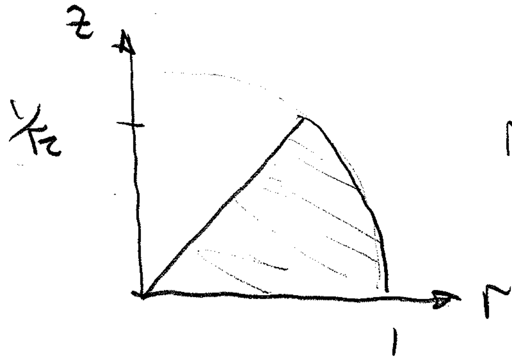
$$V = \int_{\theta=0}^{2\pi}\int_{r=0}^{1/\sqrt{2}}\int_{z=0}^{r} r\,dz\,dr\,d\theta + \int_{\theta=0}^{2\pi}\int_{z=0}^{1/\sqrt{2}}\int_{r=1/\sqrt{2}}^{\sqrt{1-z^2}} r\,dr\,dz\,d\theta$$

in a θ = ¢ plane



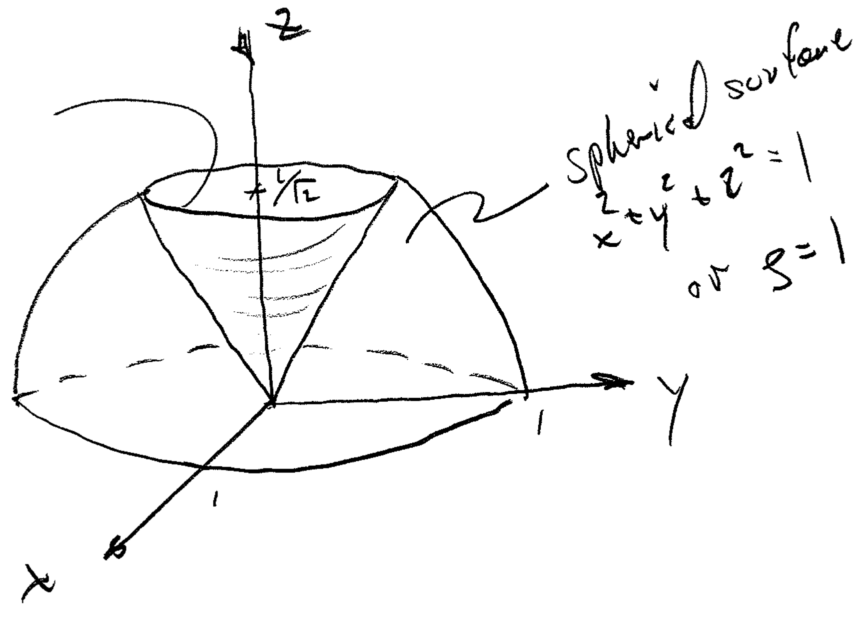


put together to get



revolve to get

cone $r = z$
is removed
also surface
$\varphi = \pi/4$



③ con't

b) $$V = \int_{\theta=0}^{2\pi} \int_{z=0}^{1/\sqrt{2}} \int_{r=z}^{\sqrt{1-z^2}} r \, dr \, dz \, d\theta$$

c) $$V = \int_{\varphi=\pi/4}^{\pi/2} \int_{\rho=0}^{1} \int_{\theta=0}^{2\pi} \rho^2 \sin\varphi \, d\theta \, d\rho \, d\varphi$$

d) $$V = \int_{\theta=0}^{2\pi} \int_{\varphi=\pi/4}^{\pi/2} \int_{\rho=0}^{1} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

e) Evaluating d)

$$V = \int_{\theta=0}^{2\pi} \int_{\varphi=\pi/4}^{\pi/2} \frac{1}{3} \sin\varphi \, d\varphi \, d\theta$$

$$= \int_{\theta=0}^{2\pi} \frac{1}{3} \cos\left(\frac{\pi}{4}\right) d\theta \quad = \frac{1}{3} \cos\left(\frac{\pi}{4}\right) \cdot 2\pi$$

$$= \frac{2}{3} \pi \frac{1}{\sqrt{2}}$$

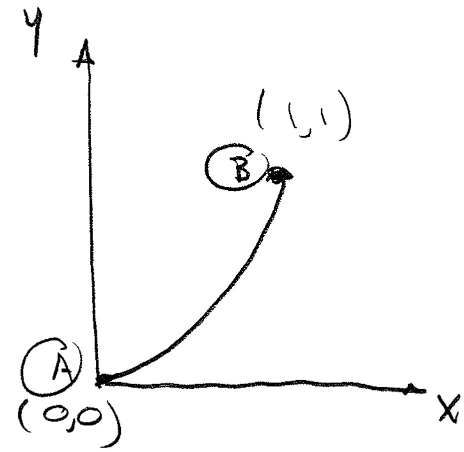
$$= \frac{\sqrt{2}}{3} \pi$$

(4)

$$\underline{r}_1(t) = t\hat{i} + t^2\hat{j} \qquad 0 \le t \le 1$$

$$\underline{V}_1 = \hat{i} + 2t\hat{j}$$

$$\underline{F} = x\hat{i} + xy\hat{j}$$



a) Flow $= \int_{A}^{B} \underline{F} \circ \hat{T}\, ds = \int_{A}^{B} \underline{F} \circ \underline{V}\, dt$

$$= \int_{t=0}^{1} \underbrace{\left(x + xy\,2t\right)}_{\text{eval on } \underline{r}} dt = \int_{t=0}^{1} \left(t + 2t^4\right) dt$$

$$= \left(\frac{t^2}{2} + \frac{2}{5}t^5\right|_0^1 = \frac{1}{2} + \frac{2}{5} = 9/10$$

b) Flux $= \int_{A}^{B} \underline{F} \circ \hat{n}\, ds = \int_{A}^{B} \underline{F} \circ \tilde{V}\, dt \qquad \tilde{V} = 2t\hat{i} - \hat{j}$

$$= \int_{t=0}^{1} \underbrace{\left(x2t - xy\right)}_{\text{eval on } \underline{r}_1} dt = \int_{t=0}^{1} \left(2t^2 - t^3\right) dt$$

$$= \left(\frac{2}{3}t^3 - \frac{t^4}{4}\right|_0^1 = \frac{2}{3} - \frac{1}{4} = 5/12$$

4 cont

c) Pick any paramet. that puts you on the curve $y = x^2$. For example...

$$\underline{r}_2(t) = t^2\,\hat{i} + t^4\,\hat{j} \qquad 0 \le t \le 1$$

then $\underline{V}_2 = 2t\,\hat{i} + 4t^3\,\hat{j}$

$$\text{Flow} = \int \underline{F}\circ\hat{T}\,ds = \int_{t=0}^{1} \underline{F}\circ \underline{V}_2\,dt$$

$$= \int_{t=0}^{1} \underbrace{\left(x\,2t + xy\,4t^3 \right)}_{\text{eval on } \underline{r}_2} dt$$

$$= \int_{t=0}^{1} \left(2t^3 + 4t^9 \right) dt = \frac{t^4}{2} + \frac{4t^{10}}{10} \Big|_0^1$$

$$= \frac{1}{2} + \frac{2}{5} = 9/10$$

the same, as expected!