CALC 3 Ex 1 Sum '08

1 a) i iv vi vii

b) $x = 0 - 2t$
$y = 0 - t$
$z = 1 + t$

c) $d = \left| \vec{PS} \cdot \frac{n}{|n|} \right|$ $S = (2,3,4)$ $\hat{n} = \hat{i} - \hat{j} + 0\hat{k}$ $|n| = \sqrt{2}$
$P = (0,0,0)$ $\vec{PS} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

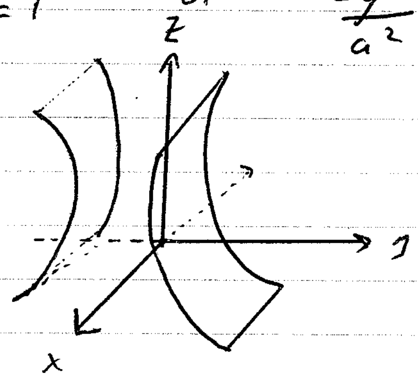
$\left| \vec{PS} \cdot \frac{\hat{n}}{|n|} \right| = \left| (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot \left(\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} + 0\hat{k} \right) \right|$

$\left| \frac{2}{\sqrt{2}} - \frac{3}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}}$

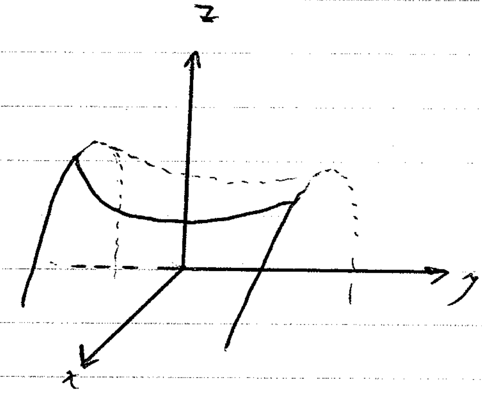
d) $\vec{b} = 4\hat{i} + 2\hat{j} + 0\hat{k}$ $\vec{a} = \hat{i} + \hat{j} + \hat{k}$
$Proj_A B = \left(\frac{A \cdot B}{A \cdot A} \right) A$
$\vec{a} \cdot \vec{b} = 4 + 2 = 6$ $\frac{6}{3}\vec{a} = \boxed{2(\hat{i} + \hat{j} + \hat{k})}$
$\vec{a} \cdot \vec{a} = 3$

2) a) Any Ellipsoid of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ where $a^2 = b^2$ and $c = \frac{a}{2}$

b) $\frac{y^2}{a^2} - \frac{z^2}{b^2} = 1$ or $-\frac{y^2}{a^2} + \frac{z^2}{b^2} = 1$



c) $4x^2 - 9y^2 + 36z = 0$
$\frac{x^2}{9} - \frac{y^2}{4} + z = 0$
$z = \frac{y^2}{4} - \frac{x^2}{9}$ hyperbolic paraboloid



3) $r(t) = 2t\,\hat{i} + t^2\,\hat{j} - \frac{t^3}{3}\,\hat{k}$

a) $v(t) = 2\hat{i} + 2t\hat{j} - t^2\hat{k}$

$|v(t)| = \sqrt{4 + 4t^2 + t^4} = \sqrt{(t^2+2)^2} = \cancel{2t}\; t^2+2$

$|v(1)| = 3$ cm/sec

b) $s(t) = \int_0^t |v(\tau)|\,d\tau = \int_0^t \tau^2 + 2\,d\tau = \frac{\tau^3}{3} + 2\tau\Big|_0^t = \frac{t^3}{3} + 2t$

$s(t) = \frac{t^3}{3} + 2t$

c) $a(t) = 0\hat{i} + 2\hat{j} - 2t\,\hat{k}$

d) $a_T = \frac{d|v|}{dt} = \frac{d(t^2+2)}{dt} = 2t$

$a_N = \sqrt{|a|^2 - a_T^2} \Rightarrow \sqrt{4 + 4t^2 - 4t^2} = 2$

e) $K = \frac{a_N}{|v|^2} = \frac{2}{(t^2+2)^2}$ or $\frac{|v \times a|}{|v|^3} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2t & -t^2 \\ 0 & 2 & -2t \end{vmatrix}}{(t^2+2)^3} =$

$\left|(-4t^2 + 2t^2)\hat{i} - (-4t)\hat{j} + 4\hat{k}\right| = \left|-2t^2\hat{i} + 4t\hat{j} + 4\hat{k}\right| =$

$\sqrt{4t^4 + 16t^2 + 16} = \sqrt{(2t^2+4)^2} = 2t^2 + 4 = 2(t^2+2)$

$\frac{2(t^2+2)}{(t^2+2)^3} = \frac{2}{(t^2+2)^2}$

f) $\frac{\begin{vmatrix} 2 & 2t & -t^2 \\ 0 & 2 & -2t \\ 0 & 0 & -2 \end{vmatrix}}{|v \times a|^2} = \frac{0\,|\;| - 0\,|\;| - 2\begin{vmatrix} 2 & 2t \\ 0 & 2 \end{vmatrix} = -8}{2t^2 + 4 = 2(t^2+2)}$ $\quad [2(t^2+2)]^2$

$\boxed{\frac{-2}{(t^2+2)^2}}$

4) surfaces

C	A	D
F	B	E

5) $\vec{r}(t) = t\hat{i} + \cos(t)\hat{j} - \sin(t)\hat{k}$

$\vec{v}(t) = \hat{i} - \sin(t)\hat{j} - \cos(t)\hat{j}$

$|\vec{v}| = \sqrt{1+1} = \sqrt{2}$

$$\hat{T} = \frac{1}{\sqrt{2}}\hat{i} - \frac{\sin(t)}{\sqrt{2}}\hat{j} - \frac{\cos(t)}{\sqrt{2}}\hat{k}$$

$$\hat{T}' = 0\hat{i} - \frac{\cos(t)}{\sqrt{2}}\hat{j} + \frac{\sin(t)}{\sqrt{2}}\hat{k}$$

$$|\hat{T}'| = \sqrt{\frac{\cos^2(t) + \sin^2(t)}{2}} = \frac{1}{\sqrt{2}}$$

$$\hat{N} = 0\hat{i} - \cos(t)\hat{j} + \sin(t)\hat{k}$$

$$\vec{B} = T \times N = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1/\sqrt{2} & -\sin(t)/\sqrt{2} & -\cos(t)/\sqrt{2} \\ 0 & -\cos(t) & \sin(t) \end{vmatrix} = \frac{-\sin^2(t)}{\sqrt{2}} \frac{-\cos^2 t}{\sqrt{2}}\hat{i} - \frac{\sin(t)}{\sqrt{2}}\hat{j} + \left(\frac{-\cos t}{\sqrt{2}}\right)\hat{k}$$

$$\vec{B}(t) = -\frac{1}{\sqrt{2}}\hat{i} - \frac{\sin(t)}{\sqrt{2}}\hat{j} - \frac{\cos(t)}{\sqrt{2}}$$

$$\vec{B}\left(\frac{3\pi}{2}\right) = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + 0\hat{k}$$

$$\vec{r}\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2}\hat{i} + 0\hat{j} + K$$

$$A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$$

$$-\frac{1}{\sqrt{2}}\left(x - \frac{3\pi}{2}\right) + \frac{1}{\sqrt{2}}(y) = 0$$