Name: ______________________________

APPM 2350 Exam #2 Summer 2008

Be sure to include your name and a grading table on the front of your blue book. You must work all of the problems on this exam. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please clearly indicate the start of each new problem. Good luck!

1. (20 points) Consider $f(x, y) = \dfrac{x^2 - y^2}{xy}$

   (a) Where is $f(x, y)$ continuous?

   (b) What is $\lim\limits_{(x,y)\to(0,0)} f(x, y)$?

   (c) Find all first partial derivatives of $f(x, y)$. Simplifying your answers will be helpful.

   (d) If $x(\theta) = \sin(2\theta)$ and $y(\theta) = \cos(2\theta)$, find $\frac{df}{d\theta}$.

2. (20 points) Let $f(x, y) = \sin(x - y)$.

   (a) Determine the linearization of $f(x, y)$ at the origin.

   (b) Bound the error in this approximation if $-\frac{1}{5} \leq x \leq \frac{1}{5}$ and $-\frac{1}{5} \leq y \leq \frac{1}{5}$.

   (c) What is the quadratic approximation to $f(x, y)$ at the origin?

3. (20 points) A large heated storage room is shaped like a rectangular box and has a volume of 8000 cubic meters. Because warm air rises, the heat loss per unit area through the ceiling is five times as great as the heat loss through the floor. If the heat loss through the four walls is three times as great as the heat loss through the floor, determine the room dimensions that will minimize heat loss and thus minimize the heating costs. **In order to receive full credit** for this question, you must demonstrate the solution minimizes the heat loss by evaluating the objective function at another point on the constraint-surface.

4. (20 points) Let

$$f(x,y,z) = \frac{(y+1)^2(z+1)^2}{2} + e^{yz}x^2$$

be a function defined on $\mathbb{R}^3$.

(a) Compute the gradient of $f(x,y,z)$.

(b) Compute the directional derivative of $f(x,y,z)$ in the direction of the vector $\mathbf{w} = -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ at the point $P(1,0,0)$.

For parts (c) and (d), also consider the unit sphere centered at the origin be described as the level surface

$$g(x,y,z) = x^2 + y^2 + z^2 = 1$$

(c) If a **unit vector**

$$\hat{\mathbf{u}} = u_1\hat{\mathbf{i}} + u_2\hat{\mathbf{j}} + u_3\hat{\mathbf{k}}$$

exists in the plane tangent to the unit sphere at the point $P(1,0,0)$, what has to be true about the relationship between $\hat{\mathbf{u}}$ and $\nabla g|_P$?

(d) Is there a unit vector in the plane tangent to the sphere at the point $P(1,0,0)$ such that the directional derivative $D_{\hat{\mathbf{u}}}f$ is equal to $\frac{1}{2}$?

5. (20 points) Let the electric potential, $P(x,y)$, on a circular region be given by

$$P(x,y) = 100 - 2x^2 - y^2$$

At $t = 0$, assume a negative test charge is placed inside this region at the point $(-4,4)$. At each instance, this test charge will move in a direction so that the increase in electric potential is a maximum. In addition, it moves with a **speed** equal to the absolute value of the change in electric potential at its instantaneous position.

(a) In what **direction** does the charge initially move and with what **speed**?

(b) Draw several level curves of $P(x,y)$ in the xy-plane. In addition, sketch the path of the charge.

(c) If the velocity of the charge is expressed as $\mathbf{v}(t) = \frac{d}{dt}\mathbf{r}(t) = \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dx}{dt}\hat{\mathbf{j}}$, find the parameterization of $\mathbf{r}(t)$.