Name: ______________________

APPM 2350 | Exam #3 Version B | Summer 2008

Be sure to include your name and a grading table on the front of your blue book. You must work all of the problems on this exam. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please start of each new problem **on a new page**. Good luck!

1. (24 points) Integration!

   (a) Evaluate

   $$\int_{\pi/6}^{\pi/2}\int_0^1 \frac{(1-r^2)\cos\theta}{\sin\theta}\, r\, dr\, d\theta$$

   (b) The following double integrals represent the area of a region in the xy-plane.

   $$\int_{-1}^{0}\int_0^{\sqrt{1+x}} dy\, dx + \int_0^1\int_{\sqrt{2x}}^{\sqrt{1+x}} dy\, dx$$

   i. Sketch the region of integration.
   ii. Rewrite the area of the region as one double integral.
   iii. Evaluate the one double integral.

   (c) Evaluate

   $$\iiint_R \sqrt{x^2+y^2+z^2}\, dV$$

   where R is region bounded below by the cone $\phi = \frac{\pi}{3}$ and above by the sphere $\rho = 2$.

   (d) Evaluate

   $$\int_0^2\int_0^{\sqrt{2-x^2}}\int_0^{\sqrt{x}} y\, z\, dy\, dz\, dx$$

2. When studying the spread of an epidemic, we can model the likelihood a disease will spread to an individual. Suppose that our model indicates a person has over a 50% chance of contracting a disease if they are within 1 mile of 100 other infected individuals.

   Assume the population density of a circular city centered at the origin is approximated by

   $$\delta(x,y) = \frac{50}{(1+x^2+y^2)^2} \quad \text{individuals per square mile}$$

   Will a person located at the center of the city have a 50% chance or greater of contracting the disease?

3. (18 points) Describe the set of points satisfying the following cylindrical/spherical equations using one or more equations in rectangular coordinates. **In addition** sketch the described set in xyz-space.

   (a) $r^2 + z^2 = 2z$ (b) $\phi = \sin \pi$ (c) $\rho = \dfrac{\csc \phi}{\cos \theta + \sin \theta}$

4. (30 points) Consider the double integral

$$\iint_S \frac{x + 2y}{\sqrt{x - y}}\, dA$$

where S is the parallelogram bounded by the lines

$$x - y = 0 \quad x + 2y = 0 \quad x - y = 1 \quad x + 2y = 2$$

   (a) Sketch the region in the xy plane.

   (b) Define the transformation from the xy-plane to the uv-plane as $u = x + 2y$ and $v = x - y$. Using this transformation, sketch the image of S in the uv-plane and **label** the bounding curves. Call the image region of S in the uv-plane R.

   (c) Determine the Jacobian of the transformation from the region R in the uv-plane to the region S in the xy-plane.

   (d) Evaluate the double integral

$$\iint_S \frac{x + 2y}{\sqrt{x - y}}\, dA$$

   as a double integral over R in the uv-plane using results from parts (a), (b) and (c).

5. (12 points) The integral $I = \displaystyle\int_0^\infty e^{-x^2}\, dx$ is both improper and impossible to evaluate using only elementary expressions from Calc 2. However, if we also define $I = \displaystyle\int_0^\infty e^{-y^2}\, dy$, then I^2 can be expressed as a double integral. Evaluate the double integral to solve for I.

97% of all statistics are made up.