

Name: \_\_\_\_\_

APPM 2350

FINAL Exam

Summer 2008

Be sure to include your name and a grading table on the front of your blue book. Show ALL of your work and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, a wrong answer with no work will receive no credit, and an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, cell phones, calculators, or electronic devices of any kind are NOT permitted. Please start of each new problem **on a new page**. Good luck! **There are three sections on this exam. Please read each section carefully or you will end up doing way more work than you need to.** Note that this exam is worth 150 points.

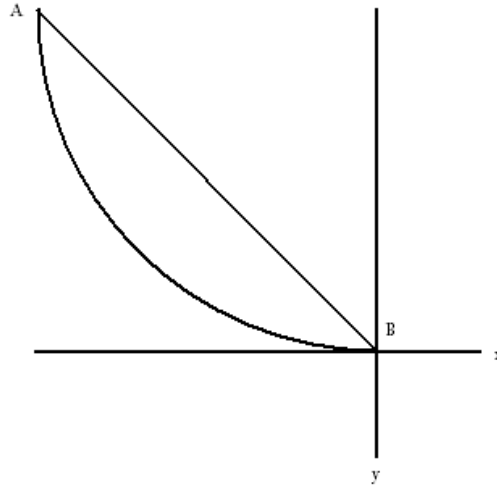
### SECTION A: WORK QUESTIONS 1-3

1. (20 points) Find the flow along the curve given by  $\mathbf{r}(t) = \langle 2 \cos(t), 2t, -3 \sin(t) \rangle$  with  $0 \leq t \leq \pi$  in the vector field  $\vec{F}(x, y, z) = \langle 20x^3z + 2y^2, 4xy, 5x^4 + 3z^2 \rangle$ .
2. (20 points) Find the flux of  $\vec{F}(x, y, z) = \langle x + \cos(y), y + \sin(z), z + e^x \rangle$  through the closed surface bounded by  $z = 0$ ,  $y = 0$ ,  $y = 2$  and  $z = 1 - x^2$ .
3. (20 points) A liquid is swirling around in a cylindrical container of radius 2 oriented along the  $z$ -axis and bounded by the planes  $z = 0$  and  $z = 10$ . The fluid motion is described by the velocity field  $\vec{F}(x, y, z) = -y\sqrt{x^2 + y^2} \hat{\mathbf{i}} + x\sqrt{x^2 + y^2} \hat{\mathbf{j}}$ . Find the circulation around the curve defining the intersection of the top of the cylindrical container and its rounded sides (i.e. upper surface of the cylindrical container).

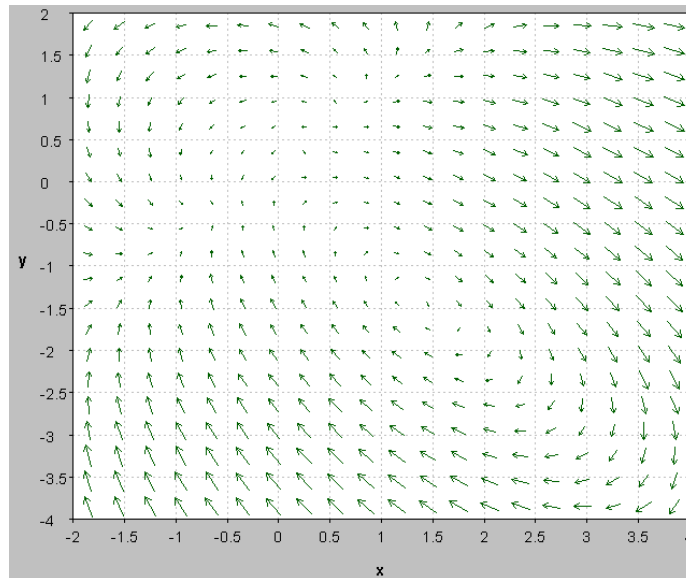
### SECTION B: WORK 4 OF THE NEXT 6 QUESTIONS

4. (15 points) Two unrelated questions.
  - (a) If  $\vec{v} = \langle 1, 3, -1 \rangle$  and  $\vec{w} = \langle -1, 2, -2 \rangle$ , find the vector projection of  $\vec{v}$  onto  $\vec{w}$ .
  - (b) Find the distance between the parallel planes  $3x - y - 2z = 6$  and  $3x - y - 2z = -2$ .
5. (15 points) Consider the surface given by  $z = 2x^2 + 6y^2$ .
  - (a) Find an equation for the tangent plane at the point  $(1, -1, 8)$ .
  - (b) Find an equation for the normal line at the point  $(1, -1, 8)$ .

6. (15 points) Two paths connect point  $A(-1,1)$  to point  $B(0,0)$  as in the attached picture.
- (a) Path 1 is a straight line. Write a parameterization for that line.
- (b) Path 2 is a quarter circle. Write a parameterization for the quarter circle.



7. (15 points) Consider the graph of *the gradient of an electric potential function*. You place a negatively charged test particle at  $(1, -2)$ . Sketch the particle and its approximate path. Indicate the direction of motion.



8. (15 points) Locate and identify all critical points of the function  $f(x, y) = 2x - x^2 + 2y^2 - y^4$ .

9. (15 points) Compute the limit

$$\lim_{(x,y) \rightarrow (1,2)} \frac{3(y-2)(x-1)^2}{(x-1)^2 + (y-2)^2}$$

**SECTION 3: WORK EITHER QUESTION 10 or 11**

10. (30 points) Consider the volume of the solid bounded above by  $x^2 + y^2 + z^2 = 4$  and below by  $z = \sqrt{\frac{x^2}{3} + \frac{y^2}{3}}$

- (a) Sketch the solid object represented by this integral.
- (b) Set up but do not evaluate  $\iiint_D dz dy dx$ .
- (c) Change the order of integration to  $dz dr d\theta$ .
- (d) Change the order of integration to  $d\rho d\phi d\theta$ .
- (e) Evaluate one of the integrals in part (b), (c), or (d).

11. (30 points) Consider the surface  $S$  given by  $S = \{(x, y, z) \text{ such that } x^2 + y^2 + z^2 = 1 \text{ and } x, y, z \geq 0\}$ . The following subproblems will guide you through computing the flux through this surface using a line integral.

- (a) Sketch the surface  $S$  and the curve  $C$  that bounds the surface.
- (b)  $C$  can be expressed as the union of 3 smooth curves  $C = C_1 \cup C_2 \cup C_3$ .

$$\begin{aligned} C_1 : \vec{r}(t) &= \cos(t) \hat{\mathbf{i}} + \sin(t) \hat{\mathbf{j}} + 0 \hat{\mathbf{k}} \\ C_2 : \vec{r}(t) &= 0 \hat{\mathbf{i}} + \cos(t) \hat{\mathbf{j}} + \sin(t) \hat{\mathbf{k}} \\ C_3 : \vec{r}(t) &= \sin(t) \hat{\mathbf{i}} + 0 \hat{\mathbf{j}} + \cos(t) \hat{\mathbf{k}} \\ &0 \leq t \leq \frac{\pi}{2} \end{aligned}$$

- (c) Let  $\vec{G} = xy^2 \hat{\mathbf{i}} + yz^2 \hat{\mathbf{j}} + x^2z \hat{\mathbf{k}}$ . Compute the curl of  $\vec{G}$ .
- (d) Define  $\vec{F} = \nabla \times \vec{G}$ . Use the definition of flux and Stoke's Theorem to show the flux of  $\vec{F}$  through  $S$  can be written as a line integral.
- (e) Evaluate the line integral in part (d).

**Why did the chicken cross the Mobius strip?  
To get to the same side.**