APPM 2350 **Exam 1, Wednesday, September 24** **Fall 2008**

On the front of your bluebook, write: (1) your name, (2) your recitation session number (see the table to the right for reference), and draw a grading table, as shown in the margin to the right. Apart from for Problem 1 (for which you should give only the answers), you must show all your work in your blue book, and BOX your final answers (A correct answer with no relevant work shown might not receive any credit, while an incorrect answer with some correct work might receive partial credit). With the exception of a 1 sheet (2 page) handwritten crib sheet, no text book, notes, or calculators are permitted. Please start each new problem on a new page of the bluebook.

Recitation	Time	Room	TA
011	9:00 - 9:50 pm	MUEN E130	Galanthay, Ted
012	10:00 - 10:50 am	MUEN E130	Galanthay, Ted
013	11:00 - 11:50 am	MUEN E130	Rasca, Anthony
014	12:00 - 12:50 pm	MUEN E130	Rasca, Anthony
015	1:00 - 1:50 pm	MUEN E130	Alexander, Zach
016	2:00 - 2:50 pm	MUEN E130	Alexander, Zach
021	9:00 - 10:50 am	ECCR 110	Sirisubtawee, Sekson
022	10:00 - 10:50 am	ECCR 118	Sirisubtawee, Sekson
024	11:00 - 11:50 am	ECCR 133	Woods, Chris
025	12:00 - 12:50 pm	ECCR 110	Woods, Chris
026	1:00 - 1:50 pm	ECCR 118	Villavert, John
027	11:00 - 11:50 am	ECCR 137	Liu, Kuo
031	9:00 - 9:50 am	ECCR 116	Boorn, Jason
032	10:00 - 10:50 am	ECCR 110	Chestnut, Stephen
033	11:00 - 11:50 am	ECCR 110	Chestnut, Stephen
034	11:00 - 11:50 am	ECCR 118	Kochen, Michael
035	1:00 - 1:50 pm	ECCR 110	Kochen, Michael
036	12:00 - 12:50 pm	ECCR 137	Liu, Kuo

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1. (30 points) For each of the statements below, tell if it is TRUE or FALSE (meaning not always true):

a. $\theta = \sin^{-1}(|\boldsymbol{A}\times\boldsymbol{B}\,|/(|\boldsymbol{A}|\,|\boldsymbol{B}|))$ ($\boldsymbol{A},\boldsymbol{B}\neq\boldsymbol{0}$, θ is the angle between $\boldsymbol{A}$ and $\boldsymbol{B}$).

b. $\theta = \cos^{-1}((\boldsymbol{A}\cdot\boldsymbol{B}\,)/(|\boldsymbol{A}|\,|\boldsymbol{B}|))$ (same condition, notation as above)

c. $(\boldsymbol{A}\cdot\boldsymbol{B})\cdot\boldsymbol{C} = \boldsymbol{A}\cdot(\boldsymbol{B}\cdot\boldsymbol{C})$

d. $(\boldsymbol{A}\times\boldsymbol{B})\cdot(\boldsymbol{C}\times\boldsymbol{D}) = (\boldsymbol{D}\times\boldsymbol{C})\cdot(\boldsymbol{B}\times\boldsymbol{A})$

e. $(\boldsymbol{A}\times\boldsymbol{A})\cdot\boldsymbol{A} = 0$

f. $(\boldsymbol{A}\times\boldsymbol{B})\cdot\boldsymbol{A} = 0$

g. $(\boldsymbol{A}\times\boldsymbol{B})\cdot\boldsymbol{A} = \boldsymbol{B}\cdot(\boldsymbol{A}\times\boldsymbol{B})$

h. $\dfrac{d}{dt}[c^2(t)\,\boldsymbol{r}(t)] \;= c(t)[2c'(t)\,\boldsymbol{r}(t) + c(t)\,\dfrac{d\boldsymbol{r}(t)}{dt}]$

i. $\dfrac{d}{dt}[\boldsymbol{r}(t)\times\boldsymbol{s}(t)] \;= \dfrac{d\boldsymbol{r}(t)}{dt}\times\boldsymbol{s}(t) + \dfrac{d\boldsymbol{s}(t)}{dt}\times\boldsymbol{r}(t)$

j. $\boldsymbol{A}\times(\boldsymbol{A}\times(\boldsymbol{A}\times(\boldsymbol{A}\times(\boldsymbol{A}\times\boldsymbol{B})))) \;= \boldsymbol{0}$

2. (15 points) Consider a position vector $\boldsymbol{r}(t)$ with a velocity vector that satisfies $|\boldsymbol{v}(t)| = 1$.

a. Find the arc length of $\boldsymbol{r}(t)$ from $t = t_0$ to $t = t_1$.

b. Show that the acceleration $\boldsymbol{a}(t)$ is orthogonal to the velocity $\boldsymbol{v}(t)$.

c. We can decompose the acceleration into a sum of two components: $\boldsymbol{a}(t) = a_T(t)\boldsymbol{T} + \boldsymbol{a}_N(t)\boldsymbol{N}$, where $a_T(t)\boldsymbol{T}$ is the component of acceleration in the tangent direction and $a_N(t)\boldsymbol{N}$ is the component of the acceleration in the normal direction. Give simple expressions for $a_T(t)$ and $a_N(t)$.

Please turn over $\Longrightarrow$

3. (20 points) Jimmy J., a surveyor, measures three points $A(0,0,1)$, $B(3,0,0)$, and $C(0,2,0)$ on a planar slope S.

a. Write a standard equation for the plane S and verify that points A, B, and C satisfy this equation.

b. There is an object at point $D(0,0,0)$ that Jimmy's company wants to reach by drilling. Find a point E on plane S that is closest to point D (so they can drill the shortest path).

c. The path from D to E has ground that is hard to dig through. Jimmy determines that digging from somewhere on the line segment from point A to point B will be much easier on the equipment. Find the point F on the segment $\overline{AB}$ that is closest to point D.

d. The company also needs to lay a straight pipeline that heads southwest-to-northeast ($\boldsymbol{i}$ components and $\boldsymbol{j}$ components are equal) and stays the same distance below the ground. Give an equation for a line that never crosses plane S, goes through point D, and has $x(t) = y(t)$ for any value of the parameter t.

4. (20 points) Find $\boldsymbol{T}$, $\boldsymbol{N}$, $\boldsymbol{B}$, the curvature κ, and the torsion τ for the plane curve $\boldsymbol{r}(t) = (2t+3)\,\boldsymbol{i} + (5-t^2)\,\boldsymbol{j}$.

5. (15 points) Determine if the following limits exist and, if they do, give their values:

a. $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{(\tan x)^2}$ b. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2}$ c. $\displaystyle\lim_{(x,y)\to(1,-1)} \frac{xy+x+2y+2}{y+1}$

<u>Some potentially useful formulas:</u>

$\text{Proj}_A \boldsymbol{B} = \left(\frac{\boldsymbol{A}\cdot\boldsymbol{B}}{\boldsymbol{A}\cdot\boldsymbol{A}}\right)\boldsymbol{A}$,

Distance from a point S to a line through P in direction $\mathbf{v}$: $d = \frac{|\boldsymbol{PS}\times\boldsymbol{v}|}{|\boldsymbol{v}|}$,

Distance from a point S to a plane through P with normal $\mathbf{n}$: $d = \frac{|\boldsymbol{PS}\cdot\boldsymbol{n}|}{|\boldsymbol{n}|}$.

T, N, B system: $\boldsymbol{T} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|}$, $\boldsymbol{N} = \frac{d\boldsymbol{T}/dt}{|d\boldsymbol{T}/dt|}$, $\boldsymbol{B} = \boldsymbol{T}\times\boldsymbol{N}$;

Curvature and Torsion: $\kappa = \left|\frac{d\boldsymbol{T}}{ds}\right| = \left|\frac{d\boldsymbol{T}}{dt}\right|\cdot\frac{1}{|\boldsymbol{v}|} = \frac{|\boldsymbol{v}\times\boldsymbol{a}|}{|\boldsymbol{v}|^3}$, $\tau = -\frac{d\boldsymbol{B}}{ds}\cdot\boldsymbol{N} = \begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix} / |\boldsymbol{v}\times\boldsymbol{a}|^2$